Reply to Gao's "Comment on "How to protect the interpretation of the wave function against protective measurements"

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Abstract

Shan Gao (Gao 2011) recently presented a critical reconsideration of a paper I wote (Uffink 1999) on the subject of protective measurement. Here, I take the occasion to reply to his objections.

1 Introduction

In 1993, Aharonov, Anandan and Vaidman (Aharonov, Anandan & Vaidman 1993) introduced a new type of measurement procedures in quantum theory, which they dubbed "protective measurements". In these procedures, one can measure, under certain conditions and for a specific set of states, the expectation value of an arbitrary observable of an individual system. Remarkably, such expectation values are obtained while avoiding the subsequent entanglement of the states of the system and the apparatus, even if the system was initially not in an eigenstate of the measured observable. In this respect, the protective measurement is very different from the more well-known von Neumann measurement procedure.

Aharonov, Anandan and Vaidman (AAV) attributed this feature of protective measurements to a physical manifestation of the wave function of the system. They claim that by means of these measurements one can directly observe the wave function (or quantum state) of an individual system, and conclude from this that this quantum state should be given an ontological interpretation: if it is possible to observe the state of an individual system, it must correspond to a real property of this system.

In 1999, I published a paper that challenges these claims (Uffink 1999). In particular I argued, first, that the conditions employed in the notion of protective measurements imply that only observables that commute with the measured system's Hamiltonian can be measured protectively. Secondly, I proposed an alternative interpretation of protective measurements that explain the remarkable achievements of protective measurements without relying on a physical manifestation of the wave function of a single system. Thirdly, I argued that a thought experiment proposed by AAV to illustrate their claims as providing empirical evidence of delocalization in the state which is a superposition of two localized states for a single particle, fails to provide that alleged evidence.

Gao's critique (Gao 2011) challenges all of these three claims in my paper. Indeed, (i): Gao observes an alleged "deadly error" in a proof given to substantiate my first claim. Further (ii): He criticizes the alternative interpretation of protective measurements offered in my paper, and (iii): He finds problems with my analysis of the thought-experiment mentioned above. I will address these three concerns in consecutive order in the sections below. In brief, I admit, concerning point (i), that the proof in section 4 of (Uffink 1999) is not correct. However, the error committed is far from deadly. In section 2, I will provide an improved proof to substantiate my claim that only observables that commute with the system's Hamiltonian can be measured protectively. In sections 3 and 4, I argue that Gao's objections (ii) and (iii) miss the point.

2 Must the observable measured protectively commute with the Hamiltonian?

In the course of (Uffink 1999), I wanted to analyze the consequences of the various approximations made by the proponents of protective measurements in their consideration of the evolution of the composite state of an object system and a measurement device. Therefore, I decided to compare the actual evolution operator U by the approximation U_{app} which resulted from the application of first-order perturbation theory and the adiabatic theorem, and formalize the assumption that the approximation is "good" as a limit condition. The claim I made in the paper is that these conditions imply that the observable O commutes with the Hamiltonian H_S.

For details of the argument I refer to (Uffink 1999). After some manipulations which are not disputed by Gao, this limit condition was put in the form

$$\lim_{\tau \longrightarrow \infty} \langle \phi_m | e^{i\tau H_S + pO)} H_S e^{-i(\tau H_S + pO} | \phi_n \rangle = E_n \delta_{mn} \tag{1}$$

Here, O is the observable being measured, H_S is the system Hamiltonian, $|\phi_n\rangle$ and E_n denote the (non-degenerate) eigenstates of H_S and their eigenvalues respectively. Furthermore, the limit should hold for any choice of m and n and almost all values of $p \in \mathbb{R}$.

In (Uffink 1999), I wrote that this limit condition is equivalent to

$$\lim_{\tau \longrightarrow \infty} e^{i\tau(E_m - E_n)} \langle \phi_m | e^{ipO} H_S e^{-ipO} | \phi_n \rangle = E_n \delta_{mn} \tag{2}$$

and used this formulation to substantiate the claim that O commutes with H_S.

However, Gao correctly observed that this argument seems to presuppose the equality $e^{i\tau H_S + pO} = e^{i\tau H_S} e^{ipO}$, which however, as is well-known, only holds in general when O and H_S commute. Hence, the proof seems to be circular because it presupposes what it sets out to proof. Now, it is hard for me to reconstruct today why I thought this argument convincing 13 years ago. I admit that in retrospect, Gao is completely right in pointing out an omission in this proof and I am grateful for that.

However, although Gao's observation implies that the proof offered for my claim was incomplete, it does not imply that the claim is invalid. Thus, Gao's judgment that my proof suffers from a "deadly error", or is "doomed to failure" is premature. In fact, it is not too difficult to provide an argument which avoids the presupposition of what needs to be proved altogether. The rest of this section is devoted to provided this proof.

Since, by assumption, the condition (1) should hold for almost all values of $p \in \mathbb{R}$, we can choose a value of p which is very small but not equal to zero. Now define

$$A := \frac{\tau}{p} H_S + O \tag{3}$$

so that (1) becomes:

$$\lim_{\tau \longrightarrow \infty} \langle \phi_m | e^{ipA} H_S e^{-ipA} | \phi_n \rangle = E_n \delta_{mn} \tag{4}$$

By the Baker-Campbell-Hausdorff theorem, we have

$$e^{ipA} H_S e^{-ipA} = \sum_{k=0}^{\infty} \frac{(ip)^k}{k!} H_k \tag{5}$$

where

$$H_0 = H_S, \qquad H_1 = [A, H_S], \qquad H_2 = [A, [A, H_S]], \qquad H_k = [A, H_{k-1}]. \tag{6}$$

Now, the basic strategy behind the present proof is that by taking $|p|$ to be very small, we can ignore all but the first few terms in the expansion (5). However, one needs to be careful in executing this strategy, since A, as defined by (3) itself depends on p, and although for very small values of $|p|$, the factor $\frac{(ip)^k}{k!}$ rapidly goes to zero for increasing k, the factor H_k might blow up for small $|p|$, e.g. by containing terms proportional to p^{-k} to upset this rapid decrease. However, this worry can be laid to rest by explicit inspection.

In view of (3) and (6) we get

$$\begin{aligned} H_0 &= H_S \\ H_1 &= [A, H_S] = [O, H_S] \\ H_2 &= [A, [A, S]] = [A, [O, H_S] = \frac{\tau}{p} [H_S, [O, H_S]] + [O, [O, H_S]] \end{aligned} \tag{7}$$

and so on. In general, H_k will only contain terms proportional at most to $p^{-(k-1)}$, and therefore, the higher order terms in the expansion (5) (i.e. the terms where k is large) will indeed become negligible when $|p|$ is very small.

In view of this argument, and assuming $|p|$ to be very small, we only need to investigate the first two terms of the series expansion (5), upon substitution in (4). For $k = 0$, we get

$$\langle \phi_m | H_0 | \phi_n \rangle = \langle \phi_m | H_S | \phi_n \rangle = E_n \delta_{mn} \tag{8}$$

which already equals the desired limit under $\tau \longrightarrow \infty$ in (4). This means that condition (4) can only hold if the contributions from the terms with $k \geq 1$ in (5) vanish in the limit $\tau \longrightarrow \infty$. For $k = 1$, we have a contribution to the series expansion of the left-hand side of (4)

$$ip\langle\phi_m|H_1|\phi_n\rangle = ip\langle\phi_m|[O, H_S]|\phi_n\rangle \tag{9}$$

Note that this term does not depend on τ, and hence will not be affected by the limit $\tau \longrightarrow \infty$. It follows that the condition (4) can only hold for the chosen value of p if we have

$$\langle\phi_m|[O, H_S]|\phi_n\rangle = 0 \tag{10}$$

In other words, the matrix elements of $[O, H_S]$ vanish for any values of m and n in the complete orthonormal basis $\{|\phi_n\rangle\}$, and this implies that $[O, H_S] = 0$, i.e. O commutes with the Hamiltonian H_S.

To sum up, I admit the proof in (Uffink 1999) was incomplete, but the omission pointed out be Gao is not, as he put it, a "deadly error", nor is the proof "doomed to failure". Instead, the proof can be completed, and I believe that the claim that O commutes with H_S is actually correct, when the approximative assumptions employed in a protective measurement are granted.

3 The alternative interpretation of protective measurements

In order to explain the surprising features of protective measurements in a way that I found more easily accessible than the interpretative claims of AAV, I proposed the view that the procedure does not actually measure the observable O, but the related observable $\tilde{O}$, defined as

$$\tilde{O} = \sum_n P_n O P_n \tag{11}$$

where $P_n = |\phi_n\rangle\langle\phi_n|$ denotes a projector on the eigenstates of the Hamiltonian H_S.

Gao finds this interpretation problematic, however. First of all, he argues that whereas a measurement of $\tilde{O}$ does not lead to entanglement (or collapse), such effects cannot be avoided in a measurement of O due to the finite value of the measuring time τ, even though these effects can be made arbitrarily small.

Here, I believe that Gao did not fully appreciate the logic of my argument. I argued that a protective measurement of O, assuming that the approximative assumptions imposed by AAV, were in place, could also be explained as a measurement of $\tilde{O}$. Of course, one can back down on this assumption and say that these assumptions (e.g. $\tau \longrightarrow \infty$) are only approximations and that if we assume finite τ, the procedure sketched as a protective measurement of O differs in various respects from a proper measurement of $\tilde{O}$. This is fine by me. However, in that case, the procedure sketched is not a proper protective measurement of O either. Thus, the argument that my interpretation of a protective measurement fails to be proper in cases where the AAV interpretation of the protective measurement fails to be proper too, can hardly be taken as an objection against my alternative interpretation. The assumption that the approximations employed by AAV hold is equally essential to my alternative interpretation as it is to the original AAV interpretation.

The second argument Gao offers against my interpretation is that a measurement of the observable $\tilde{O}$, as defined in (11) requires full *a priori* knowledge of the system's Hamiltonian. I fail to see the relevance of this consideration. We are dealing here with an interpretation of a physical theory, not with a theory about knowledge. The observable $\tilde{O}$ is well-defined, regardless of whether we know the Hamiltonian H_S. We do not need *a priori* knowledge of the Hamiltonian, we only need to know that it exists, (and is non-degenerate) to infer the existence of $\tilde{O}$. Perhaps, in a practical case, we might wish to measure an observable O protectively without full knowledge of the system's Hamiltonian H_S. In such a case, my alternative interpretation would say that, under the approximations employed, one is actually measuring an observable $\tilde{O}$ which may be (partly) unknown because (say) the projectors P_n are unknown. But I fail to see why this should count as an objection to the proposed interpretation. We can apply quantum theory of measurement if we do not *a priori* know the state of the system, we can equally apply it when we do not know *a priori* exactly which observable is being measured. The interpretation I offered is completely neutral with respect to the information or knowledge an experimenter might actually have.

4 The thought experiment

Gao also criticizes my analysis of AAV's thought experiment. I have to defer a description of this thought experiment to the original literature. Briefly, this experiment concerns a charged particle which could be in either of two boxes L or R, and which has localized states $|\phi_L\rangle$ and $|\phi_R\rangle$, but could also be in a superposition of these two states, e.g.

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}|\phi_L\rangle \pm |\phi_R\rangle \tag{12}$$

The thought experiment proposed by AAV concerns a protective measurement of the observable

$$O = |\phi_L\rangle\langle\phi_L| - |\phi_R\rangle\langle\phi_R| \tag{13}$$

and argued that, when the system is the state $|\phi_+\rangle$, a protective measurement of O yields a value $\langle O\rangle_+ = 0$, which was taken as evidence that the state $|\phi_+\rangle$ is delocalized, i.e. a superposition of the two localized states $|\phi_L\rangle$ and $|\phi_R\rangle$ and thus provide a physical manifestation of the quantum state of a single system.

However, I argued in (Uffink 1999) that the very same measurement procedure, performed when the system was in either of the localized states $|\phi_L\rangle$ or $|\phi_R\rangle$ would yield the same result, and that the result thus provides no evidence for delocalization in the superposition of these states.

Gao does not dispute the details of this argument, and argues that "at first sight this argument seems reasonable, However, it is not difficult to find its problem by a careful analysis." To find this problem, he puts forward a new condition:

"The key is to realize that in order to measure the state of a measured system, e.g. whether the system is in a delocalized state or not, the state must be protected before the measurement. This is a basic requirement of protective measurement. "

This requirement, which states that any measurement should be a protective measurement is entirely new. Even AAV only proposed the concept of protective measurements as an additional tool in quantum theory, but never claimed that they were the only legitimate kind of measurements. Of course, applying this new condition to my analysis, Gao finds faults: The same procedure that constitutes a protective measurement of O when performed on $|\phi_+\rangle$ does not qualify as a protective measurement of O when performed on $|\phi_L\rangle$ or $|\phi_R\rangle$. Again, this is fine by me. In fact, my (Uffink 1999) anticipated this remark, as is acknowledged by Gao.

But although Gao and I seem to agree on the above point, we apparently differ in what conclusions to draw from this shared point. In my view, the conclusion is simple: if we agree that if the proposed measurement procedure will give the same experimental result, regardless of whether the state of the system is localized (i.e. $|\phi_L\rangle$ or $|\phi_R\rangle$) or not localized (e.g. $|\phi_\pm\rangle = \frac{1}{\sqrt{2}}|\phi_L\rangle \pm |\phi_R\rangle)$, we should not regard this experimental result as empirical evidence for delocalization, or as a physical manifestation of the delocalized wave function, as AAV proposed.

In Gao's view, as I understand it, the procedure proposed to measure O is a protective measurement when carried out on a delocalized state like $|\phi_\pm\rangle = \frac{1}{\sqrt{2}}|\phi_L\rangle \pm |\phi_R\rangle)$, but not when the same procedure is carried out on states like $|\phi_L\rangle$ or $|\phi_R\rangle$, and by the new requirement he adduced, the procedure should not qualify as measurement at all. He argues that such procedures do not tell us that states like $|\phi_L\rangle$ or localized or not, and thus do not support my conclusions.

However, in response I should mention, first, that the question whether the proposed procedure would qualify as a protective measurement or not was never part of my assumptions. I based my conclusion that the proposed procedure in this thought experiment does not provide evidence for delocalization of the quantum state on the argument that the very same procedure would yield the same outcome, regardless of whether the state was localized or not. This seems to me to violate a basic issue of what one can take to be empirical evidence for any phenomenon at all, even in a thought experiment. Secondly, in his discussion of measurement, Gao seems to drift from the (common) concept of measuring an observable to the (uncommon) idea of measuring a state, and proposed a new requirement on the latter idea. I do not agree on the additional requirement proposed by Gao, but readily admit that when I said "the very same measurement procedure" I intend the concept of measurement to be understood as measurement of an observable rather than that of a state. So, to me, measurements of the observable O, whether it is is carried out on states like $|\phi_L\rangle$ or $|\phi_R\rangle$ or on $|\phi_\pm\rangle$ are instances of the same measurement procedure, i.e. a measurement of O, irrespective of whether they all qualify as protective measurements of a state. Thus, I fail to see how Gao's considerations would affect my argument.

Furthermore, I do not see how Gao's objections on my analysis of this thought experiment relate to the question of whether the result of such a procedure will tell us whether or not a state like $|\phi_L\rangle$ is localized or not. In my view, the state $|\phi_L\rangle$ is localized by its very definition. We do not need a measurement to assure us of this fact, and if the result of any procedure performed on this state should or should not positively reaffirm us about this, there should be no reason to doubt what we have assumed by definition.

Finally, Gao also expresses puzzlement about my attitude when I wrote the paper (Uffink 1999). He argues that my views are due to biassed philosophical opinions, and that an alternative view, in which we would be able to decide univocally on the interpretation of the wave function, would be much more exciting and satisfying.

Of course, fully I agree with Gao that such an alternative view would be much more desirable. However, apart from the hot aspirations we might all have concerning the interpretation of quantum theory, we also need the cool breeze of critical analysis before we step forward. It is only in this spirit, rather than by a preconceived biassed attitude, that I wrote the paper criticized by Gao.

References

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