The Fundamental Arrow of Time in Classical Cosmology

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Abstract

The aim of this paper is to show that a new understanding of fundamentality (section 2), can be applied successfully in classical cosmology and is able to improve a fundamental understanding of cosmological time asymmetries.

In the introduction (section 1), I refer to various views from different literature. I begin by arguing against various arguments, provided in favour of the view that the directedness of time is not a fundamental property of nature, (see for example Price [(1996)]) in section 1.1.

Next, in section 1.2, I refer to some suggestions for defining the arrow of time in cosmology via the behaviour of entropy. In addition, I argue that the various types of such approaches cannot explain the occurrence of a *fundamental* arrow of time in cosmology; I believe that this argument is mostly acknowledged in the literature. Moreover, in section 1.3, I broaden my scope in order to formulate approaches in the context of theories of a multiverse and hyperbolic curved spacetimes as well.

After that overview of different approaches, I provide a definition of my understanding of fundamentality (section 2).

To verify the effectiveness of this new understanding, I present an example to show that it can be applied in classical cosmology in sections 3.1–3.5. Finally, in section 4 I conclude by regarding the effects of adding the new understanding of fundamentality to classical cosmology.

1 Views from different literature

Before discussing the reports of various literatures, it is necessary to clarify two important points: First, the used terminology regarding the terms 'time asymmetry', 'directedness of time', 'arrow of time' and 'time direction' and second, my understanding of the terms

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‘fundamental’ or ‘non-fundamental’ in the context of the terms ‘time asymmetry’, ‘directedness of time’, ‘arrow of time’ and ‘time direction’.

In this paper, I will understand the term ‘time asymmetry’ as the most general term. An equation, a function, a physical model or a whole physical theory, will be called ‘time asymmetric’, if the change of the sign of the time coordinate(s) in the equation, function, physical model or theory provides some sort of change in the equation, the value of the function or the physical content of the model or theory.

Furthermore, the term ‘directedness of time’ is used to describe the particular situation, in which the time coordinate(s) under consideration, in the context of an equation, a function, a physical model or a theory, behaves asymmetrically regarding the change of the sign of the time coordinate(s) on *every* time point.

Thus, a time asymmetry can provide the directedness of a time coordinate in a given context of an equation, a function, a physical model or a theory, if the coordinate, in this context, behaves asymmetrically on every time point.

Additionally, I will use the term ‘arrow of time’ to describe the situation that, in the context of a physical model or theory, the time coordinate(s) in the theory has the property of directedness.

Thus, the directedness of time provides an arrow of time, if the theoretical or formal context in which the directedness of time is given is a whole physical model or a theory.

The term ‘time direction’ is used if, in a given context of an equation, a function, a physical model or a whole theory, the directedness of time is given and additionally the sign of the time coordinate(s) is fixed for intrinsically rezones. This means that the time direction is not provided by time directedness or the arrow of time alone. The arrow of time is given by the directedness of time but the orientation of the arrow is not fixed by that property. (For a deeper motivation and consideration of the used terminology regarding the term ‘time direction’ see for example Earmann [(1974)] or Price [(2011)]).

In this paper I will focus on the arrow of time in the sense from above and only on the property of the directedness of time. But if, for example, the cosmic time coordinate contains an arrow of time, it is possible that, even if this coordinate has ‘only’ the property of time directedness, the physical and inter-theoretical connections to other fields in physics, apart from cosmology, yields a time direction of other time coordinates (for example proper times). But this is not in the scope of this paper and a sufficient analysis of this possibility would require an analysis of the inter-theoretical connection between the directedness of the cosmic time coordinate and the proper time coordinates in other fields of modern physics.

So, I shall come to the first sketched understanding of fundamentality. In section 2, I precisely define fundamentality through mathematical conditions and the associated motivation. Before moving ahead, I shall explain which properties, according to me, an arrow of time and a time asymmetry should possess for it to be called fundamental.

First, a fundamental arrow of time should be based on the fundamental properties of the theory, which is assumed as fundamental in a given physical description of nature.

Second, a fundamental arrow of time and a fundamental time asymmetry should be global, i.e. at least in our particular spacetime, the arrow should not change its (perhaps conventional chosen) direction.

Thus, on the basis of these properties, I shall define fundamentality through mathematical conditions in section 2. But, before, I will argue that the prominent accounts, at least those that I am aware of, cannot satisfy the mentioned conditions for a fundamental time arrow.

1.1 The time-symmetric view

In this subsection, I briefly examine the suggestions of Price ([1996]) regarding time symmetry[2] because the arguments that favour a time-symmetric view do not appear plausible or compelling; thus, the search for a fundamental arrow of time is not doomed to fail, at least not owing to the arguments made by Price.

Price starts his cosmological analysis with the observation that the early state of the universe is special in one interesting way: the universe near the Big Bang is smooth (Price [1996]).[3] A matter distribution consisting of black holes would be much more likely than a smooth

[2] The following argumentation is focused on an interpretation of Price [(1996)] in which his arguments are considered to be arguments *for* a time symmetric understanding of cosmology. If the arguments from Price [(1996)] are considered 'only' as arguments *against* some well-known statistical attempts to the cosmological time arrow and not *for* a time symmetric understanding, my view is in perfect agreement with Price [(1996)].

[3] However, on this point, a crucial question arises that is important for understanding Price's suggestions: It is not clear what the word 'early' (which Price uses many times; see, for example, Price 1996 pp. 79 and 80) means in this context. Price does not seem to refer to a proper time period with respect to the constituents of the universe when he uses the phrase 'early state of the universe'. Instead, it seems that he refers to a) cosmic time or b) the geometrical fact that these states of the universe are 'near' (according to a metric such as the FRW metric) the Big Bang. To make this interpretation of Price plausible, we need a definition of the distance between a spacetime point p (or the singularity) and a three-dimensional spacetime region R (a state of the three-dimensional universe). However, there are many possible ways in which we could define this distance, for example, taking the nearest spacetime point p' of a region R and calculating the distance between p and p' according to the metric. Of course, we can also construct many definitions of distance for this task. Such definitions would be more or less plausible, but they all define what it means to say that 'a spacetime region is *near* a spacetime point or the Big Bang'. It seems fair to assume that Price refers to the most acceptable meaning of 'early'. That is, it seems most unproblematic to define his use of the term 'early' as 'near the Big Bang according to a metric and a plausible definition of distance between a spacetime point p and a spacetime region R'.

distribution if classical gravity is the dominant force, as assumed in classical cosmology. Thus, according to classical thermodynamics, Price argues that this fact shows that the ‘early’ universe has very low entropy.

Now, Price ([1996], p. 78) continues to argue that entropic behaviours (as well as all other properties), cannot provide a fundamental time arrow in classical cosmology. His arguments are based on the fact that all statistical considerations, which in fact yield the second law of thermodynamics, are also valid in the reverse time order. Thus, the time asymmetry of entropic behaviour is based on the initial low-entropy conditions of the Big Bang (see also Albert [2000]). Therefore, Price ([1996], pp. 81–99) argues that according to entropic behaviour (or other statistical reasoning’s), a closed universe with low-entropy boundary conditions in the ‘future’ is time symmetric and the low entropy boundary in the ‘further’ is as likely as the one in the ‘past’, which seem to be given in our actual world. Additionally (see Price [1996], pp. 95–96), Price argues that we are not concerned with whether our particular spacetime is closed or open, but with whether a closed spacetime with symmetric boundary conditions is possible given the laws of classical cosmology.

‘*This point [the possibility of open spacetime geometry] is an interesting one, but it should not be overrated. For one thing, if we are interested in whether the Gold universe [a kind of a time-symmetric closed spacetime] is a coherent possibility, the issue as to whether the actual universe recollapses is rather peripheral. […] Of course, if we could show that a recollapsing universe is impossible, given the laws of physics as we know them, the situation would be rather different*’ (Price [1996], p. 95).

Thus, Price seems to argue that if the existence of such a universe is possible given the laws of classical cosmology, there is no reason to assume that time is directed in a fundamental sense. Even if our particular spacetime would have boundaries that yield an entropic time asymmetry; this would not affect a fundamental time direction because the direction, in such a spacetime, would be given by (perhaps accidental) boundary conditions.

On this point it seems necessary to mention that, perhaps, Price [(1996)] can be understood in a weaker sense (see also footnote 1). In my description of Price arguments above, I had assumed that his claim is to argue for the implausibility of a fundamental time asymmetry in the theory of classical cosmology. If, in contrast to that assumption, the main claim of the consideration is to argue against statistical methods for defining a cosmological arrow or time, than his claim is in perfect agreement with my view. The conflict only arises if the arguments from Price [(1996)] are taking as arguments against the plausibility of a fundamental time arrow in general.

Regarding this interpretation of Price my critique can be outlined very briefly. Price mainly argues that in a possible closed spacetime, there would be no physical parameter that distinguishes a Big Bang from a Big Crunch (and thus could be used to define an time arrow). This is grounded on the assumption that the scale factor (Price calls it the radius of the universe), which behaves symmetrically in a closed spacetime, is the only fundamental property to distinguish time directions. If this assumption would be true, the possibility of closed spacetimes, given general relativity, would in fact show that no fundamental property can be used to distinguish between a ‘Big Bang’ and a ‘Big Crunch’. Thus, time would be symmetric in the physical description. But his crucial assumption, which according to me is wrong, seems to be that the scale factor (or the radius) is the only basic property that could be used to distinguish between the two singularities in a closed spacetime, given the standard theories of classical cosmology (see Price ([1996] pp. 86-111). Thus, I will argue that this assumption is not plausible and other geometrical properties (not statistical considerations) of spacetime apart from the scale factor must be considered for defining a fundamental time direction.

Thus, I conclude that if it is possible to define spacetime properties that are as basic as the scale factor but independent of them, Price’s arguments are no longer plausible, and thus, the cosmological time arrow could be fundamental.

Nevertheless, this brief investigation was necessary to show that the search for a fundamental time arrow is not doomed to fail on the grounds of Price’s arguments, if there are other fundamental properties of spacetime apart from the scale factor. But this, in fact, seems the case according to modern cosmological models. I will come back to this point more precise in section 3.

However, before I define ‘fundamentality’ with respect to time asymmetries, I briefly examine some approaches to the arrow of time in classical cosmology in order to show that the suggestions that I am aware of cannot define a fundamental time arrow in classical cosmology.

1.2 Entropy-based approaches

In this subsection I shall show that various entropy-based approaches cannot define a fundamental arrow of time in classical cosmology, that is, the time directedness in these approaches is not given by the basic properties of cosmology.

In this section, I focus on approaches that define the arrow of time in cosmology via time-asymmetric behaviour of entropy. More precisely, the future direction in such approaches is given by the time direction in which entropy increases according to the second law of thermodynamics. Of course, statistical mechanics show that most types of entropy also would (theoretically) increase in the time-mirrored direction. Thus, it is necessary to set some boundary conditions for the past, specifically, that the universe has low entropy in the past (see, for example, Albert [2000]).

However, approaches that focus on the temporal behaviour of entropy can be subsumed under (at least) two different topics, as analysed by Price [(2002)]. To make this point clear, in this subsection I only focused on attempts to base the cosmological time asymmetry on the entropy behaviour during a time evolution of the three dimensional universe. My claim is, in full agreement with Price [(2002)], to underline the point that such approaches are unable to provide an explanation for the occurring of a cosmological time arrow. Moreover, I consider an third kind of account, which is not captured by the analysis from Price [(2002)] and which, I think, is advocated in a very adequate way by Ćirković and Miloševic-Zdjelar [(2004)]. I will argue against this 'Acausal-Anthropic' attempt to the entropy behaviour and defending the result from Price [(2002)] that, given the actual theories in physics, the entropy behaviour is not a valid feature to yield the directedness of time.

Regarding the three mentioned types of entropy based attempts:

1. Causal–general approaches: This class of approaches seeks an explanation of the low-entropy state near the Big Bang by fundamental physical laws and an associated dynamic explanation for the specialness of the 'early' state.

But, no current physical theory can explain the specialness of the early universe only by invoking dynamic laws. All approaches that I know refer to boundary conditions of some type (such as the past hypothesis of Albert [2000]). It seems that with our current knowledge of physics, we cannot formulate approaches that deduce an arrow of time from dynamic laws (see also Wald [2006]). Thus, causal-general approaches seem unsuccessful in deducing a time direction or even the directedness of time (in a fundamental way) in classical cosmology as long as boundary conditions are not understood as fundamental properties. Given modern physical theories, it seems in fact that boundary conditions are not understood as fundamental properties. I also believe that many philosophers and physicists are aware of this situation and agree well with this view, and hence, this point is not discussed in more detail.

2. Acausal–particular approaches: This class of approaches describes the specialness (or the low entropy) of states near the Big Bang by the existence of boundary conditions.

As mentioned above, boundary conditions should not be understood as fundamental properties of physics, at least in modern physical theories. Thus, neither of these two approaches explains the occurrence of a fundamental (in some reasonable sense) arrow of time in classical cosmology.

However, as mentioned earlier, some authors have argued for another possibility (for example, Ćirković and Miloševic-Zdjelar [2004]). They argued that some type of cosmological theory describing a multiverse (more precisely, a type that includes the existence of many cosmic domains) could provide another possibility for defining the directedness of time, because some multiverse theories (see, for example, Linde [1990]) could explain the fact that our universe has a very smooth matter distribution near the Big Bang.

In the classes of multiverse theories, we have more than one universe, where each universe can be called a cosmic domain. One of these domains is our particular universe. Each of these domains could have different initial conditions. Thus, if the occurrence of smooth states near the Big Bang has a probability of, for example $1:10^{10^{123}}$ (Penrose [1979]), the prediction of the existence of cosmic domains that include such smooth early states seems very plausible, as long as the number of cosmic domains is assumed to be much larger than the reciprocal of the probability of the occurrence of such smooth early states. This prediction is as plausible as the prediction of getting a ‘6’ at least once if a fair six-sided dice is rolled n times, where n is much larger than six.

Authors, who support this view, occasionally call it an anthropic approach (Ćirković and Miloševic-Zdjelar [2004]) because of the following reasons. At first glance, it may appear very surprising in such a theory that the observable universe belongs to this minority of domains, which are only as likely as $1{:}10^{10^{123}}$ (for example). If other domains are more likely, why do we not observe such a domain in our cosmic environment? At this point, it becomes possible to consider the anthropic principle. The answer to the question would be that the domains that are more likely cannot be observed unlike other domains because no human being can survive in such a universe in which even the existence of stars or atoms is unlikely.

Thus, the anthropic principle is not used to explain a cosmological fact. The explanation of this fact comes from the cosmological theory, independent of any anthropic considerations. The anthropic principle is only used to clarify that the fact that our particular universe belonging to the small minority is not surprising because otherwise it would not have been our particular universe. However, what is interesting is that we find a cosmological theory that explains the occurrence of certain initial conditions in a particular cosmic domain. Thus, according to such types of multiverse (and often inflation) theory, an acausal–particular approach could be understood as fundamental in the sense that the basic properties of the laws of the universe, in such theories, show that some particular boundary conditions occur and provide an time asymmetry.

Thus, in addition to causal–general and acausal–particular approaches, which were also analysed by Price ([2002]), we could consider this acausal–anthropic approach in this study. I use this name as it is used in Ćirković and Miloševic-Zdjelar ([2004]).

The approach based on the possibility of time-asymmetric behaviour of entropy in our particular cosmic domain is called, from now on in this investigation, the entropic–anthropic approach. However, upon closer examination, this time asymmetry cannot be understood as being based on the laws of fundamental physics, for the following reason.

The multiverse theories predict a very large number of cosmic domains. In addition, the time parameter, which is fundamental in this context, is a quantum parameter independent of particular cosmic times in some cosmic domains. The laws that give rise to the fundamental processes of creating different cosmic domains (which could have different cosmic times) are processes in quantum physics, which can be time symmetric according to the quantum physical time parameter.[4] The fundamental laws and mechanisms of those theories also allow many cosmic domains, which could be time symmetric in terms of their cosmic times. Thus, in such domains, the behaviour of entropy is symmetric or the value of entropy is constant apart from fluctuations; obviously, this value is also symmetric.

[4] This is true if we assume the standard interpretation of quantum physics in which, for example, complex conjugation will not change the physical meaning of a quantum dynamic equation such as the Schrödinger equation.

Thus, according to the entropic–anthropic approach, we find an explanation for the occurrence of time-asymmetric behaviour in our particular cosmic domain, but this asymmetry occurs by accident. The entropic–anthropic approach explains that it is not surprising that we find ourselves in a cosmic environment such as the observable universe; nevertheless, the temporal direction is not based on basic properties of the theory, which is treaded as the fundamental one (here quantum field theory).

Thus, we have seen that all the approaches described in this section fail to describe the existence of a time asymmetry that could reasonably be called fundamental (for some reasonable understanding of fundamentality in this context). This holds for: a) all the causal-general accounts, b) the acausal-particular accounts and c) the acausal-anthropic considerations. Thus, I will discuss another prominent approach to define an arrow of time in cosmology. This approach is independent of the behaviour of entropy.

1.3 Hyperbolic curved spacetimes

Many cosmological models of the evolution of our particular universe have a property that we have not yet discussed in this paper. Observations and theoretical work support the idea that the three-dimensional universe exhibits accelerated expansion in cosmic time (e.g. Riess et al. [1998] on supernovae observations; Barlett and Blanchard [1996] on the cosmic virial theorem; Fan, Bahcall and Cen [1997] on mass indicators in galaxy clusters; Bertschinger [1998] on large-scale velocity maps and Kochanek [1995] or Coles and Ellis [1994]). This could be described in general relativity as a large positive value of the cosmological constant. Moreover, this seems to indicate that it is plausible that the universe has an open geometry, not only because the matter and energy density are too low to overcome expansion but also because the expansion is accelerated by a ‘force’ described by the cosmological constant.

Now, the question arises whether this large value of the cosmological constant occurs accidentally or owing to some law of physics. If the accelerated expansion of the universe were the result of a fundamental law, time asymmetry could be defined regarding the direction pointing to the open end of spacetime and this time asymmetry could be based on the properties of the fundamental law. However, I must argue that this is not convincing given our current knowledge of cosmology.

Note first that the value and origin of the cosmological constant still remains unsolved if we consider a one universe picture (no many cosmic domains). Moreover, even if multiverse pictures are assumed a particular value of a cosmological constant in one cosmic domain would occur accidently in those pictures and not for fundamental reasons (see above).

Nevertheless, I briefly examine one possible account because some authors (Ćirković and Miloševic-Zdjelar [2004]) had attempted to argue that a particular account of the origin of the cosmological constant yields a special understanding of the arrow of time in cosmology.

This account is that the cosmological constant is a result of vacuum polarisation. The effect of vacuum polarisation is surely very fundamental; moreover, it arises from the laws of quantum field theory (QFT).

I shall show that this assumption does not allow us to conclude that the open geometry of spacetime yields a time asymmetry based on the properties of a fundamental law. This is because the force of gravity, which opposes the cosmological constant, depends on the matter and energy density of the universe. Thus, the critical value that the cosmological constant must exceed to yield an accelerated universe in such a semi-classical model depends on the matter and energy density. This density does not seem to be determined by QFT in general. Moreover, some types of cosmological theory suggest that this density could vary among cosmic domains in the multiverse picture. If such variation is possible, it follows that the effect of the accelerated expansion of the universe could only be used to define an time asymmetry in some particular cosmic domains; However, I think this asymmetry should not be understood as fundamental, because we notice the same situation in standard classical cosmology in which a spacetime could be open or closed, given the mass and energy density. Thus, an account based on the large value of our particular cosmological constant yields the same problems as the accounts mentioned above: a closed universe is possible even if the value of the cosmological constant is assumed to be ‘large’ for fundamental rezones. As long as we prefer to call a time asymmetry a fundamental one only if the directedness of time is based on fundamental properties of the theory used in the model, Ćirković and Miloševic-Zdjelar ([2004]) cannot provide new insight on this question by considering the origin of the large value of the cosmological constant in vacuum polarisation. As mentioned, this is because a closed spacetime geometry is not ruled out (in principle) by a large value of the cosmological constant.

Thus, the fact that our particular universe or cosmic domain seems to present an accelerated expansion cannot be used to conclude that we can define a cosmological time asymmetry based on the *fundamental* properties of a used physical description.

Hence, the attempts to define a time asymmetry in cosmology, which we discussed till this point, do not yield a fundamental understanding of this asymmetry. Thus, in the following section, I present a more precise understanding of fundamentality, which allows an understanding of the time asymmetry as some kind of ‘fundamental’ property of nature according to classical cosmology. I will show more precise what I mean by ‘some kind of’ in section 3, where I try to apply my definition to classical cosmology.

2 A new understanding of fundamentality

The first step in building my argument is well known and has been analysed by many authors in both philosophy and physics. It arises from the distinction between the property of time reversal invariance and the property of time asymmetry in general. In this paper, ‘time reversal invariance' will be understood as a property of dynamical equations and 'time asymmetry' will be considered in particular as a property of the solutions of those equations. Here 'time symmetry' is given by a solution $f(t)$ if $f(t_0+t)=f(t_0-t)$ holds for one t_0, hence in particular if a solution describes closed curves in phase space. Moreover we can identify dynamical equations with physical laws and solutions to those equations with physical models that satisfy such laws.[5] Therefore, we can combine these properties in four different ways:

a) time reversal invariance and only time-symmetric solutions,
b) time reversal invariance and some time-asymmetric solutions,
c) no time reversal invariance and only time-symmetric solutions and
d) no time reversal invariance and some time-asymmetric solutions.

For combination a), it is easy to find physical examples. However, this combination is not useful if we are interested in time asymmetries.

Combination b) looks interesting because it shows that time-reversal-invariant laws (TRIL) could have time-asymmetric solutions. Simple examples are given in classical electrodynamics; nevertheless, such traditional asymmetries are not understood as fundamental time asymmetries because they occur only in some special models of the TRIL and the occurrence of such asymmetries is explained mostly by boundary conditions.

The applicability of combination c) or d) in fundamental physics is at least problematic. It seems that non-TRIL's cannot be found within the laws of fundamental physics in the standard interpretation. Hence those combinations seem applicable only in some special formulation of quantum laws, for example in some formulations of the rigged Hilbert space approach (see, for example, Bohm, Gadella and Wickramasekara (1999); Bishop (2004), Castagnino, Gadella and Lombardi (2005) or Castagnino, Gadella and Lombardi (2006)). However, it will be shown below that combination c) or d) need not be used to understand the time-asymmetries in a fundamental manner.

In the following, I shall show that combination b) indicates another plausible way to define '*fundamentality*' for time-asymmetries, based only on the structure of the solution space of physical equations.

Definition I:

Suppose L is a fundamental time-reversal-invariant dynamic equation and S(L) is the solution space with dim(S[L]) = n. I will call a time asymmetry 'fundamental' if and only if

i) There is not more than a countable collection $S_i(L)$ *of subspaces of dimensions* $m_i < n$*, such that if* $f(t) \in S(L)$ *is time reversible, then* $f(t) \in S_i(L)$ *for some i and if* $f(t) \in S(L)$ *is not time reversible, then* $f(t) \notin S_i(L)$ *for all i.*

ii) For time-asymmetric solutions $f(t) \in S(L)$*, the solution* $f(-t) \in S(L)$ *refers to the same physical world as f(t) does.*

[5] Of course, not all dynamical equations describe physical laws, but physical laws can be understood as being described by dynamical equations.

Condition ii) is important because if $f(t) \in S(L)$ and $f(-t) \in S(L)$ describe physically different worlds, we would have to explain why only one direction (+ or − sign) occurs in nature. Then, this argument would fix the time direction. However, if condition ii) holds, no such additional argument is needed.

Moreover, in order to prevent a misunderstanding, I will highlight that the *combination* of both criteria is important. Criteria i) stipulates: That 'almost all' solutions of a fundamental law containing an intrinsic time asymmetry. But this alone is not sufficient for creating a plausible notion of fundamentality in the context of time asymmetries. This can be seen considering many time reversal invariant dynamic equation. Often, the particular solutions of such an equation are time irreversible and hence, intrinsically time asymmetric. But the time mirrored solutions are also solutions of the dynamic equation. Thus, the intrinsic time asymmetry of one particular solution cannot be used in order to define fundamental time asymmetries.

But if criteria i) is combined with criteria ii) the combination provides a plausible concept of fundamentality. This is because, criteria ii) adds that the intrinsic time asymmetric solutions (both $f(t) \in S(L)$ and $f(-t) \in S(L)$) are describing the same time asymmetry. Thus, the set of both solutions describes the same physical time asymmetry as one particular solution alone, because both solutions, even if formal distinguished, describing the same physical world and hence, the same physical time asymmetry.

Thus, to summarize the improvement of the new definition of fundamentality regarding the actual debate in the context of the time's arrow; I think the improvement is given by the fact that the intrinsic time asymmetry of the set of almost all solutions of a fundamental physical law can be sufficient (if both criteria are fulfilled) for defining a fundamental time asymmetry. This possibility of understanding the time asymmetry and the directedness of time as a consequence of the structure of the set of solutions as sufficient for defining also an arrow is, I think, overlooked in the discussion. Note also that such a fundamental time asymmetry is not only given in our actual world but, if both criteria are fulfilled, in the set of physical possible 'worlds'.

In fact, this possibility becomes plausible only with the definition of 'fundamentality' from above. But moreover, I think, at least some examples are needed, which show that this notion of fundamentality can be applied at least to some fields of fundamental physics. This will be the content of section 3.

So, in conclusion, a law with a set of solutions that satisfies conditions i) and ii) of definition I will always (except for some subspaces with dimension $m_i < n$) give rise to a time asymmetry in nature. Therefore, it seems clear that a time asymmetry in nature that satisfies definition I can (and perhaps should) be called fundamental (here it is important to notice that the structure of the solution space of the law is, of course, given by the law. Thus if the law is fundamental the structure of the solution space is given for fundamental rezones). In Fig. 1, I try to illustrate the structure of the solution space of a TRIL that fulfils condition i) of definition I.

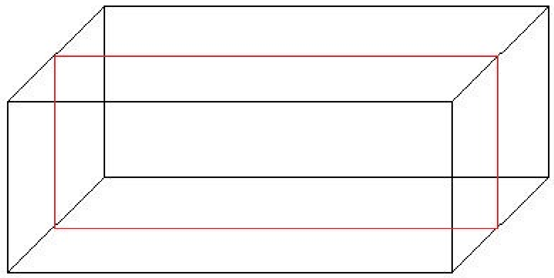

Fig. 1 The cuboid illustrates part of a three-dimensional solution space. This solution space includes both time-symmetric and time-asymmetric solutions. All solutions on the surface indicated by the red rectangle are time symmetric. This fragmentation is possible only because the solution space of time-symmetric solutions has only two dimensions (where the whole solution space has dimension three) and this fact is provided by the first part of definition I. Therefore, almost all solutions of the dynamic equations include an intrinsic time asymmetry.

The structure of the solution space of a fundamental law is clearly a fundamental property. In fact, it is based only on the fundamental law. Thus, it seems quite reasonable to say that time asymmetries are fundamentally imbedded in a theory if the solution space of the fundamental equations fulfils i) and ii).

Summing up, a fundamental physical law, which has an associated solution space which fulfils criteria i) and ii), is, I think, sufficient for the constitution:

a) Regarding the context of a physical theory, which enclosed the law, the directedness of time is a fundamental property, regardless of the actual direction.

b) The orientation of the time's arrow (from which part of the asymmetry to which part), thus the direction, is not physical (criteria ii)), even if the directedness is understood as a fundamental property of the time coordinate(s).

But note; the orientation of a time's arrow could be a consequence of the fundamental time directedness of a time coordinate. The consideration of this possibility requires the consideration of the inter-theoretical connection between the time coordinate, for which the directedness is fundamentally given in an context (e.g. cosmic time in cosmology), and the time coordinate, in (perhaps another) context, which can be effected and orientated from the directedness of the first coordinate. But this is not the main question of this paper and should not be considered in more detail here.

Instead, at this point the question arises of whether we can find a law-like equation with a set of solutions in existing physical theories, which satisfying criteria i) and ii). One such example, which I shall show, can be found in classical cosmology. Monecief ([1975] or [1976]) and also Castagnino, Lara and Lombardi ([2003a] and [2003b]) have shown that the Einstein equation produce a similar situation when we make some additional assumptions.

3 A new fundamental approach

In this section, I present my suggestion regarding the problem of defining fundamental time asymmetries in the context of classical cosmology. I show that an arrow of time can be deduced from the geometrical structure of spacetime and I will demonstrate under which conditions such an arrow could be understood as fundamental (according to definition I in section 2).

This section is organized as follows:

1. A brief discussion regarding the situation in classical cosmology according to the Einstein's field equation.

2. I present two conditions on spacetime that are necessary to support my argument. These conditions are motivated by methodological and physical considerations, as described later.

3. The set of solutions of the Einstein equation that satisfy these conditions (point 1) contain time-symmetric and time-asymmetric solutions. I show that under some physically motivated conditions, the space of the time-symmetric solutions has dimension $m < n$, where n is the dimension of the total space of solutions that satisfy the same conditions. Thus, condition i) of definition I is satisfied.

4. Furthermore, I show that the space of solutions that satisfies the assumed conditions also satisfies criteria ii) of definition I.

Thus, in almost all[6] solutions, we could say that time asymmetry and the directedness of cosmic time is a generic property of spacetime geometry under the assumed conditions.

5. I reject a possible objection to my conclusion.

Thus, in short, I will argue that in the solutions space of the Einstein equation, under some reasonable conditions, all time reversible solutions are contained in subspaces of measure zero. Thus, the generic solutions of the Einstein equation are not time reversible.

Additionally, given the definition of fundamentality, I shall argue that the resulting structure of the solution space of the Einstein equation is sufficient to establish a) a fundamental time

[6] In the sense given by i) in definition I.

asymmetry, b) a fundamental difference between ‘past’ and ‘future’ (were the labels are conventional, but the difference is substantial) and c) a fundamental arrow of time in classical cosmology.

3.1 The Situation in Classical Cosmology

Given the aim of this study it seems appropriate to investigate the initial situation in classical cosmology. Most parts of this investigation are already done in Section 1. Nevertheless it seems helpful to specify the situation. The prominent Einstein Equation (3.1)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{4\pi G}{c^4} T_{\mu\nu} \tag{3.1}$$

were $R_{\mu\nu}$ describes the Ricci-Tensor, $g_{\mu\nu}$ the metrical tensor, R the Ricci-Scalar, Λ the cosmological constant, G the gravitational constant, c the vacuum-velocity of electromagnetic waves and $T_{\mu\nu}$ denotes the energy-momentum-tensor, is time reversal invariant. But, as outlined in Section 2, this fact has no relevance for the investigation of the associated solution set of (3.1). The fundamental time asymmetry in the structure of the solution set, as I will argue, provides, as particular explications of this fundamental asymmetry; the directedness of cosmic time in almost all models in classical cosmology if two crucial conditions are assumed.

The didactics of the argument will be as follows: First, I shall argue that, in a most simple toy model, the suggested understanding of fundamentality in the context of time asymmetries is applicable. Additionally, I will argue that the applicability is not provided from the simplification of the first used toy model; instead there represent a generic property of the solution space of (3.1) if two additional conditions are assumed. Those conditions will be motivated in the following.

3.1 Additional conditions

The first of the two conditions that I will introduce in my analysis is that when investigating the solution space of the Einstein equation, I consider only those solutions for which cosmic time can be defined.

This assumption, of course, seems to be critical. Solutions of the Einstein equation, which are not time orientable, exist; hence, cosmic time cannot be defined in such spacetimes. At first glance, it appears that an approach that assumes the definability of cosmic time could not lead to a fundamental time asymmetry, since this assumption is not motivated by general relativity.

I will show why I believe that the assumption of the definability of cosmic time is acceptable. In classical cosmology (according to general relativity), the only time coordinates that appear at a fundamental level of description are the proper times of different elementary physical systems. Consider the simple example of two parallel world lines. The proper times according to these world lines could be synchronized, but their directions (i.e. the distinction between past and future semi light cones) are generally independent of each other. The assumption that is needed, if the directions of proper times on different world lines could, in principle, be connected, is the time orientability of spacetime. Thus, if we try to deny this assumption in order to achieve greater generality, we cannot discuss the direction of a time coordinate in general but only the direction of proper times regarding to one world line. The reason is the existence of possible spacetime geometries that represent possible solutions of the Einstein equation, which are not time orientable. This violates the requirement that the direction of a fundamental arrow of time should not change in one spacetime (section 1).

Moreover, note that in addition to the assumption of time orientability, we must assume the definability of cosmic time (which implied the time orientability). This is because otherwise we have no time parameter that allows the conception of a time asymmetry for more than one world line.

Thus, to make this point clear, not surprisingly my suggested application to classical cosmology cannot create a concept of the directedness of time that fulfils condition i) of definition I without this additional assumption. But, it shows that, if the notion of time directions for more than *one world line* is possible (which means that the condition is fulfilled), general relativity provides a ‘fundamental’ time asymmetry.

Thus, in summary, I claim to show that my suggestions in section 2 produce the following result:

By assuming the possibility of producing even a hypothetical sense of a time asymmetries valid for more than one world line, we find that classical cosmology reveals that a fundamental time asymmetry exists and that the directedness of cosmic time is given for fundamental rezones, given the restrictions described above.

The second additional condition is that the dynamics of spacetime as a whole has more than one independent dynamic variable. This seems plausible on the basis of physics. The point is that a classical toy model that includes only dynamic variations in the scale factor cannot describe the dynamics of the energy and matter content of spacetime. The dynamics of the matter and energy content is usually described by the dynamic behaviour of an additional matter field. Here the dynamic behaviour of the matter field is not determined only by the behaviour of the scale factor. Thus, in the simplest case of a scalar matter field, we need at least two independent dynamic variables, not just the scale factor, in order to describe the dynamics of spacetime. As mentioned in section 1.1, if we assume that the scale factor is the only dynamic variable in a fundamental level (see, for example, Castagnino, Lara and Lombardi [2003a], and Price ([1996])) it follows that spacetime is generically time symmetric on a fundamental level. Thus, all solutions of the dynamic equation for such a spacetime are time reversible. This situation changes in a more physical model that includes more dynamic variables in addition to the scale factor. This also shows that the critic on the account from Price ([1996]) (or at least on one possible understanding of his arguments) from section 1 is based on physical considerations. This consideration can be sketched by the need to describe the dynamics of the energy and matter content of spacetime, which is impossible in a model that has only the scale factor (or the radius in the terminology from Price ([1996])) as a fundamental quantity.

However, in the next subsection, I shall show in detail how we can define a fundamental arrow of time in classical cosmology by using the conditions described above.

3.2 Symmetric spacetimes

To define a fundamental time asymmetry, I analyse the types of spacetimes, which allow the definition of cosmic time, that are time *symmetric* regarding to cosmic time. I will show that such spacetimes belong to a subset of solutions having a lower dimension than the entire set of solutions (according to a dynamic equation that describes spacetime as a whole). This fulfils part i) from definition I. Moreover, I will also show that conditions ii) of definition I is satisfied by the solution space. Results of this form, I think, are well acknowledged in the literature (for a much more detailed discussions, see for example Moncrief [(1975) or (1976)] or Castagnino, Lara and Lombardi [(2003a) or (2003b)]. Here, the discussion is restricted to a simple toy model, which contains only two dynamical variables in the Hamiltonian equation. But I shall argue that this restricted analysis is *sufficient* for a generalisation, which will show that a fundamental time asymmetry as well as a fundamental arrow of time is a generic property of classical cosmology. Thus, another way of describing the claim of this paper is that I will show that this well acknowledged point is, combined with the new concept of fundamentality, able to create a fundamental understanding of the cosmological time asymmetry.

In order to do so note that the most open spacetimes are time asymmetric regarding to cosmic time. This is because we can define the time arrow of cosmology in an open spacetime according to the asymmetric behaviour of the scale factor.

Thus, we will not consider open spacetimes in this section even if they seem to yield the right topology for our particular universe. In the context of classical cosmology, such spacetimes are time asymmetric with respect to cosmic time and we seek the origin of time-symmetric spacetimes in classical cosmology.[7] Thus, to examine the mathematical origin of time symmetry with respect to cosmic time, there is no need to consider open spacetimes.[8]

Therefore, I shall concentrate on closed spacetimes. According to singularity theorems (Hawking and Penrose [1970]; Hawking and Ellis [1973]), such spacetimes, at least in classical cosmology, have just one maximum in the scale factor. This class of spacetimes *could* be time symmetric. Hence, we concentrate on such spacetimes to demonstrate the mathematical origin of cosmic time symmetry.

Consider, for simplicity, a simple case where the dynamics of spacetime is described by the scale factor $a(t)$ and a scalar matter field $\phi(t)$, which depend on cosmic time t.

In Hamiltonian mechanics, the crucial dynamic equations (and thus the Hamiltonian) depend on the dynamic variables and their first derivatives in cosmic time t. Thus, in our example, we have four arguments in the Hamiltonian, $a(t), \frac{da}{dt}, \phi(t), \frac{d\phi}{dt}$. Analytical mechanics always allows us to describe one of these variables as a function of others and which variable is chosen to depend on others is just a matter of description. Thus, for simplicity, I choose $a(t) = f\left(\frac{da}{dt}, \phi(t), \frac{d\phi}{dt}\right)$, where $\frac{da}{dt}, \phi(t), \frac{d\phi}{dt}$ are now independent dynamic variables.[9]

If we try to construct a time symmetric spacetime, all the dynamic variables together must behave in a time symmetric way. According to the singularity theorems of classical cosmology, we know that $a(t)$ has just one maximum. Next, we can choose the mathematical origin of cosmic time. For simplicity, I choose this origin so that $a(0)$ is exactly the maximum value of the scale factor. Thus, $a(t)$, as a function of cosmic time, is symmetric in relation to the axis a at the point $t = 0$. From this, it is obvious that $\frac{da}{dt}$ is symmetric in relation to the point $\left(t = 0; \frac{da}{dt} = 0\right)$. However, for such a spacetime to be time symmetric, the behaviour of

[7] Note that there is one exception in classical cosmology: a static universe that has an open topology but also has time symmetry. However, we will not consider the static solution of the Einstein equation because it requires fine tuning of the cosmological constant and the energy and matter content (and distribution) of the universe. Thus, according to classical cosmology, this solution is a special type that belongs to a subspace of solutions to the Einstein equation that surely has a lower dimension than the entire solution space.

[8] Spacetimes that have an open but time-symmetric and non-static topology are open to the past and future. I do not consider them because they require a change in the value of the cosmological constant and in the context of classical cosmology; the cosmological constant is constant in cosmic time. This may change in quantum or string cosmology, but that is beyond the scope of this paper. In classical cosmology, a contracting spacetime always has a Big Crunch (see Hawking and Ellis 1973 and Hawking and Penrose 1970). Thus, in classical cosmology, spacetime cannot be open in two directions of cosmic time if spacetime is not static.

[9] The Hamilton equation is, for this case, given by $H\left(\frac{da}{dt}, \phi, \frac{d\phi}{dt}\right) = 0$.

$\phi(t)$ and $\frac{d\phi}{dt}$, together with that of $\frac{da}{dt}$, must also be symmetric. Thus, in this example, only two possible behaviours of $\phi(t)$ and $\frac{d\phi}{dt}$ yield an entire spacetime that is time symmetric. Those possibilities are given by the triplets $\left\{ \left.\frac{da}{dt}\right|_{t=0} = 0, \phi(t=0), \left.\frac{d\phi}{dt}\right|_{t=0} = 0 \right\}$, which is a symmetric solution of $\phi(t)$ with respect to the ϕ axis at $t=0$ and $\left\{ \left.\frac{da}{dt}\right|_{t=0} = 0, \phi(t=0) = 0, \left.\frac{d\phi}{dt}\right|_{t=0} \right\}$, which is a symmetric solution of ϕ with respect to the point $\left(t=0; \phi(t=0)=0\right)$. Thereby, of course, it is assumed that the matter field and the first derivative of it behaving symmetrically to the chosen origin of cosmic time, otherwise the behaviour of spacetime as a whole could never be symmetric.

Thus, all symmetric solutions can be constructed using these triplets. Hence, we can construct a subspace of time-symmetrical solutions: $span\left\{ \begin{pmatrix} \left.\frac{da}{dt}\right|_{t=0} = 0 \\ \phi \\ \left.\frac{d\phi}{dt}\right|_{t=0} = 0 \end{pmatrix}, \begin{pmatrix} \left.\frac{da}{dt}\right|_{t=0} = 0 \\ \phi(t=0) = 0 \\ \left.\frac{d\phi}{dt}\right| \end{pmatrix} \right\}$. The complete space of solutions of the dynamic equation is given by $span\left\{ \begin{pmatrix} \frac{da}{dt} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \phi \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \frac{d\phi}{dt} \end{pmatrix} \right\}$.

Thus, the time-symmetric behaviour of a spacetime appears only in a subspace of solutions having a lower dimension than the entire solution space, even if we consider closed spacetimes. This additionally makes the critics on Price [(1996)], or a possible understanding of Price [(1996)], more explicit. The result shows that even a closed spacetime can be time asymmetric independent of any statistical considerations. The meaning of this fact is illustrated in the geometrical drawing in Fig. 1 in section 2.

Therefore, in classical cosmology, assuming that cosmic time is definable and that more than one independent dynamic variable describes the dynamics of spacetime, we see that time asymmetry is a generic property. Of course, this also holds if we add more dynamic variables to the cosmological model, because the calculation in those cases would be analogue and the entire space of solutions always has a higher dimension than the subspace of time-symmetric solutions (except if we have only one dynamic variable, e.g. the scale factor, which is the case in the analysis from Price ([1996])). In particular this means that the form of the Hamiltonian can vary without changing the general conclusion. Hence this simple analysis and this well-known point seems sufficient to draw conclusions regarding the solution set of (3.1), as long as the additional conditions, motivated in subsection 3.2 are accepted. Moreover, it seems that the conclusion can be used in semi-classical cosmology because additional dynamic variables in the Hamiltonian can be used to describe for example quantum fluctuation of the metrical tensor. Thus, this conclusion depends not on the special simplifications in the cosmological toy model that is used here.

We will now examine whether condition ii) of definition I is also satisfied by the solution space in this example and argue that, combined with the suggested understanding of fundamentality from section 2, this occasional in the literature emerged result (see for example Castagninio, Lara and Lombardi (2003a) and (2003b)) shows that a) a fundamental time asymmetry can be defined in the field of classical cosmology and moreover from a philosophical point of view, b) this result could be a first step in understanding the difference between past and future in some physical theories.

3.3 Solution space

The solution space for closed spacetimes that allow the definition of cosmic time and that have physical dynamics[10] is, for mathematical reasons, built of time-mirrored pairs of functions $f(t)$ and $f(-t)$, as is the case for many other time-dependent dynamic equations. Thus, every time-asymmetric solution $f(t)$ has a paired function $f(-t)$, which is also a solution to the dynamic equation. They are intrinsically asymmetric, but the directions of the asymmetries seem to be mirrored. Certainly, we are not worried about a physical superposition of both solutions, but we could argue that the approach I have suggested so far will not explain why $f(t)$ occurs instead of $f(-t)$ or vice versa. That is, I have yet not explained why I favour, for instance, $f(t)$ instead of $f(-t)$.

This concern motivates me to choose a cosmological example to demonstrate the applicability of definition I to physical theories. Otherwise, I would have no argument for favouring one time direction and I must refer to boundary conditions (as in Albert [2000]), undiscovered laws of nature (as in Price [1996]) or accidental facts. However, in this case, I can present an argument to defend my example and the suggested understanding of fundamentality.

I claim that the pairs $f(t)$ and $f(-t)$ are physically identical; thus, they do not refer to physically different worlds.

The intrinsically time asymmetric solutions $f(t)$ and $f(-t)$ are time-mirrored functions. Now my argument has three steps and it is a kind of Leibniz argument. The steps are the following:

i) $f(t)$ does not include intrinsic properties, which are not included in the same way in $f(-t)$. This is because they are only mirrored geometrical objects (spacetimes).

[10] This phrase mean: 'is described by more than one independent dynamic variable'. Hereafter, the phrase 'physical dynamics' is used in that sense.

ii) Both are global solutions that describe spacetime as a whole. This implies that there is no time parameter (or other physical parameter) outside the geometrical objects $f(t)$ or $f(-t)$. Thus, they are not related to an outstanding circumstance.

iii) Points i) and ii) together show that two time-mirrored spacetimes $f(t)$ and $f(-t)$ do not differ in intrinsic properties *and* they do not differ in any external relationships. Thus, they describe the same physical world.

Here, it is not the case that we just cannot *distinguish* between the two solutions. Taking general relativity as the fundamental theory in classical cosmology, my argument shows that there cannot be any property that differs between the two solutions. The minus sign in front of t refers only to the notion of an assumed, but not physical absolute background time. Thus, if both the solutions $f(t)$ and $f(-t)$ are identical in all external and internal physical properties and the only difference is the time direction of the solutions in a purely mathematical and non-physical absolute Newtonian coordinate system, we can conclude that both solutions describe the same physical world according to the physical theory that is treated as fundamental (general relativity). Note also that this argument arises not because of some accidental reason for the fact that there is no background time in theory, but because there *cannot* be an absolute background time or space, which is one of the fundamental lessons of general relativity. Thus, the fact that there are no relational structures is not merely accidental in general relativity, but is a most crucial lesson that I believe should be treated seriously. Thus, Leibniz's argument seems to be in the right place here, and as mentioned earlier, the conclusion is that both solutions are identical and are only labelled differently.

Hence, the solution space is not built from *physical* time-mirrored pairs. So, the directedness of cosmic time is provided from the set of solutions of the Einstein equation, if the mentioned well motivated conditions are fulfilled.

At this point I shall point attention to another possible misunderstanding of the advocated account. One could think that the suggested attempt to the cosmological time arrow has no, or only a little significance, by invoking the following example.

Consider a many particle system in classical mechanics. Now, consider the multi particle equation of motion, including all the interactions between the particles (setting aside three body collisions and non-analytical equations). One could intuitively think that the set of solutions of this equation would not (nearly not) contain time symmetric functions (perhaps only in a subset of measure zero). But this, at least intuitively, would not allow the conclusion that we had found a fundamental arrow of time in classical mechanics. So in which sense is the suggested analysis different from this example?

At this point, I like to point out that the analogy between the classical many particle case and the mentioned cosmological case is very stingy. In the mentioned many particle case, criteria ii) is not fulfilled, except the many particle system would have no environment and additionally a definition of some sort of 'cosmic time' (based on the dynamic properties of the system alone) would be possible. If those conditions are fulfilled, given the fact that the many particle system has no environment, the whole universe would be this system and I would argue that an arrow of time is given in that example. But, in the usual and intuitively jugged case from above, were the many particle system has an environment, the situation would be rather different. Even if it is assumed that all solutions to the multi particle equation of motion would be time irreversible, criteria ii) is not fulfilled and thus, my suggested definition is not applicable to that case. Moreover, for the classical many particle system, the Poincare return, even if the Poincare time is very large, is surly going to happen for formal mathematical rezones. Thus, it is not that obvious that the solutions of the multi particle equation of motion would be time irreversible. In the case were the positions and the momenta of all the particles return the curve in phase space of the system would be closed and thus, the associated solution of the many particle equation of motion would be time reversible. In principle, this is what is going to happen in a classical many particle system according to classical mechanics. Thus, not even criteria i) is fulfilled in the intuitively judged example of the many particle system, and thus, in such a case no criteria of my suggested analysis is applicable and I would agree with the intuitive view that, in the classical many particle system, no fundamental time asymmetry is given, but still this is possible in classical cosmology.

However, there still is another objection to the argument that I have presented see Earman ([1974] or [1989]). I discuss this objection in detail in the next subsection.

3.4 The CPT objection

Earman presented a general objection against some kinds of Leibnitz arguments from the form I used in section 3.3. He showed that according to CPT symmetry,[11] it follows, from the conclusion that $f(t)$ and $f(-t)$ describe the same physical world, that the CP signs are also not physically meaningful. Thus, the signs of parity and charge would have no physical meaning. This seemed incorrect to Earman, and he concluded that the argument presented in subsection 3.3 cannot be accurate. However, there is a loophole in Earman's argument: again, we have no background outside spacetime. Here, I show in greater detail how we could use this loophole to establish a similar form of Leibniz's argument regarding the C and P transformations.

First, I must mention that there is a debate on whether to interpret parity in an intrinsic or a relational way, which that has a long tradition in philosophy (e.g. Leibniz [1686], [1714] or Kant [1768], [1770]) and also in the modern philosophy of science (see e.g. Pooley [2002]; Earman [1989] or Frederick [1991] for a modern introduction to this field). My claim is that this interesting discussion will not affect my arguments, regardless of whether a relational or an intrinsic interpretation of parity is favoured.

[11] C is the charge transformation, P is the parity transformation and T is the time transformation of a particle. These transformations switch the signs of the charge (C), parity (P) and time (T). Physics seems to be invariant under the combined transformations of C, P and T. This is the meaning of CPT symmetry.

I begin with the C transformation. The sign of a charge is clearly a conventional label. It is also clear that a positive and a negative charge in an everyday environment differ from each other. In classical electrodynamics, the direction of the field lines is reversed if we change the sign of the associated electric charge. However, without a background, the direction of the field lines has no physical sense. Again, similar to the situation with time-mirrored pair functions, the following holds:

i) The intrinsic properties of two electromagnetic fields that differ from each other only in the direction of the field lines (and thus in the sign of the electric charges) are identical, because they are only mirrored.

ii) If there are no physical surroundings, there are also no relational structures that differ between two C-mirrored pair functions.

iii) Thus, both field configurations (formally distinguished by the signs of the charges) are physically identical if we do not consider the external environment.

Outside spacetime, no electromagnetic fields exist (note that the solutions of the dynamic equation discussed here describe spacetimes as a whole); thus, we cannot define extrinsic relationships. Again, this is not accidental but a crucial lesson from general relativity. Thus, we find the same situation for the directedness of time as for the directedness of electric field lines (or magnetic field lines, which are equivalent in special relativity). Thus, we find the same situation for C as for T transformations. Moreover, I believe that Leibniz's argument is as well in place as in the former case and for exactly the same reasons.

Now I shall show that the same holds for the P transformation. The parity transformation that is used to formulate the CPT theorem (for example, Gross [1999]) is a special type of Lorentz transformation given by $P(t) = t$ and $P(x) = -x$, where t is a time coordinate and x represents space coordinates. Thus, parity transformation is the mirroring of space. Again, consider a solution of a global dynamic equation that describes the properties of spacetime as a whole, $f(t,c,p)$ (where t, c and p now refer to a time coordinate, the sign of C and the parity, respectively). The P transformation of this global solution is given by $P(f(t,c,p)) = f(t,c, -p)$. Here, the minus sign in front of p indicates that all space coordinates are mirrored. However, they are mirrored according to the notion of a former absolute (and thus not physical) coordinate system. In the context of general relativity, we clearly do not have absolute directions of space outside spacetime. Hence, again, we find the following:

i) All intrinsic properties of two spacetimes, $f(t,c,p)$ and $f(t,c, -p)$, are identical and are just mirrored in an assumed mathematical background space.

ii) No external relationships exist for the spacetimes, $f(t,c,p)$ or $f(t,c,-p)$.

iii) Thus, again, both solutions, $f(t,c,p)$ and $f(t,c,-p)$, are physically identical.

Note that this indicates nothing about the possibility that one particular spacetime itself (as one particular solution of the Einstein equation) could have the property of directedness of space (or time or electric field lines). This would mean that one spacetime can have the property that p and $-p$ differ from each other in a physical sense. And this would mean that the directedness of space is given in that particular spacetime. However, the *p*-mirrored spacetime of such a space-asymmetric spacetime has the same space asymmetry and both spacetimes are identical, because there *cannot* be an absolute space outside of spacetime that defines external relationships.

Hence, this argument could not be made to show that, for example, a relational interpretation of parity is more plausible than an intrinsic one, because in a particular spacetime, the parity transformation could have a physical meaning.

Nevertheless, we are again in the same situation as outlined for T or C transformations, and we can use the outlined Leibniz's argument to conclude that C, P and thus the CP value (in our case) have no physical meaning for global solutions of the Einstein equation.

Note that my entire argument works only for such global solutions of global dynamic equations, which describe spacetime as a whole (this is why I refer to cosmological equations in my example). In the context of particle physics, where the CPT theorem was developed, we usually discuss the transformation of particle properties or systems constructed from particles (or, in QFT, field-quanta). In this case, the P value is important, as seen by the effect of parity violation in weak interactions. However, in the case discussed above, Earman's counterargument can be rejected only for a global dynamic equation and the associated solution space of *global* solutions.

Thus, we can conclude that the solution space of a dynamic equation that describes spacetime as a whole does not consist of *physically different* pairs $f(t)$ and $f(-t)$. Therefore, we find a fundamental time asymmetry in almost all spacetime geometries and the directedness of cosmic time as a fundamental physical property in classical cosmology.

4 Conclusions

This paper has demonstrated that the view, which defines fundamentality by definition I in section 2, can be successfully applied in classical cosmology, although the definition is applicable to classical cosmology only if two additional conditions are fulfilled. The resulting situation does not yield a fundamental arrow of time in general, but it shows that if we assume that time asymmetries are possible for more than one world line (which means that two crucial conditions are fulfilled; see section 3), then general relativity provides a 'fundamental' time asymmetry. Moreover, I demonstrated that other classes of approaches to the problem are misleading, at least when attempting to define the directedness of time as a fundamental property of classical cosmology. Thus, there are also doomed to fail in attempting those strategies to the orientation of the cosmological time arrow, because there are unable to provide a fundamental understanding of the arrow itself.

Motivated by the physical analysis of Moncrief [(1975) or (1976)] or Castagnino, Lara and Lombardi [(2003a) and (2003b)], I showed that the solution space of the dynamic equation which describes the dynamics of spacetime contains almost entirely (see subsection 3.2) time-asymmetric functions. This asymmetry is necessarily embedded in the 'dynamics' of spacetime. Thus, because of the structure of the solution space, I conclude that such an arrow satisfies condition i) of definition I in section 2.

In subsection 3.3, I showed that also condition ii) of definition I is fulfilled and hence the resulting time arrow could be understood as fundamental.

Moreover, because the drawn argument is very simple and aims only at the dimension of the solution set of the Hamiltonian, this argument can be extended to a large set of possible models according to (3.1). In fact the inner structure of the Hamiltonian can vary among different forms and additional dynamic variables can be added in order to describe additional properties of (3.1) or even of semi-classical quantum cosmology (for example quantum fluctuations of the metrical tensor). So, the simple argument, explicated at a toy model and combined with the suggested understanding of fundamentality has massive impact to the understanding of time asymmetries in cosmology as long as the mentioned condition on the considered spacetimes (section 3.2) are accepted.

Thus, even if some results are well-known and occasional sound in the literature, the large impact of this results, if combined with the suggested understanding of fundamentality, seem overlooked. It is shown that such a combination makes it possible to understand the difference between two time directions in cosmology (hence regarding cosmic time) in almost all models in classical cosmology as an explication (manifested in one considered spacetime) of the fundamental time asymmetry of the solution set of the fundamental dynamic equations in classical cosmology.

I also defended this understanding against a possible objection (see subsection 3.4). In classical cosmology, I proposed that this objection can be rejected because of the global nature of the solutions of the Einstein equation (which avoids the CPT objection by using a form of Leibniz's argument; see subsection 3.4).

To summarize, with the suggested understanding of fundamentality (section 2), it is possible to construct a time arrow in classical cosmology with respect to cosmic time; even if the orientation of the time arrow is not a physical property. This arrow occurs necessarily if the condition of definability of cosmic time is assumed. This condition is motivated by the fact that it is a necessary condition if we like to construct even a self-consistent notion of the possibility of the directedness of time, which is valid for more than one world line.

The question, if by these considerations the *orientation* of time, which human beings observe in their everyday live, can be clarified, is not considered in that paper. The considerations, which are needed to clarify that question, have to be considerations in the context of the inter-theoretical connection between the directedness of cosmic time and the dependents of other time coordinates (like proper times), in other fields of fundamental physics.

But note that the suggested view from this paper attempts to shows that cosmic ‘future’ and cosmic ‘past’ are fundamentally different, even if the orientation (which corresponds to the labels ‘future’ and ‘past’) of the cosmological time arrow is not jet showed to be a physical property (which follows directly from 3.3 and 3.4).

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