What Is a Reference Frame in General Relativity?

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Abstract

In General Relativity, reference frames must be distinguished from coordinates. The former represent physical systems interacting with the gravitational system, aside from possible approximations, while the latter are mathematical artefacts. We propose a novel three-fold distinction between Idealised Reference Frames, Dynamical Reference Frames and Real Reference Frames. In doing so, the paper not only clarifies the physical significance of reference frames, but also sheds light on the similarities between idealised reference frames and coordinates. It also analyses the salience of reference frames to define local Dirac observables and to propose a physical interpretation to diffeomorphism gauge symmetry.

1 Introduction

In the ‘post-Einsteinian’ physical and philosophical literature, due to some ambiguity which can be traced back to Einstein himself, it has been customary to conflate the terms ‘reference frame’ and ‘coordinate system’, which have been used somewhat interchangeably, or at least have not been always clearly distinguished. This problem has been egregiously highlighted in its philosophical-historical components and addressed in Norton (1989) and Norton (1993).[1]

This paper analyses what a reference frame is in General Relativity (GR), expanding on existing definitions in the literature. This analysis helps to further clarify the distinction between reference frames and coordinate systems. We will show that such a distinction plays a role in the physical interpretation of diffeomorphism gauge freedom, as well as the definition of local gauge-invariant observables.[2]

The use of coordinates, or manifold points, to spatiotemporally localise quantities in GR poses two main, closely interconnected problems:

(P1) We cannot define local Dirac observables

(P2) We have an interpretation of diffeomorphism gauge as mathematical redundancies or

[1]According to Norton's analysis, ‘Einstein's coordinates’ had physical meaning, in that they were mathematical structures in $\mathbb{R}^4$ (‘Einstein's manifold’), directly representing physically possible spacetimes. Modern practice has, however, confused ‘Einstein's coordinates’ with ‘coordinate charts’ (see Section 2), here called coordinate systems, which are merely labellings of geometric structures in a differentiable manifold $\mathscr{M}$, without any physical referent. See also Gomes (2023a).

[2]Having this distinction clear also has a pedagogical role: it is good not to confuse the two terms, even if one uses them consistently.

‘descriptive fluff’ (Earman (2004))

We will show that both problems are naturally solved when not coordinates (nor manifold points) are used to localise physical quantities, but (spatiotemporal) reference frames.[3] As we will specify later (footnote 24, section 3.3), the mention of the use of manifold spacetime points is to emphasise that we do not mean to limit our proposal to the choice of a coordinate representation. In this regard, we identify the gauge group of GR with the group of active diffeomorphisms.[4]

Gauge transformations lead to redundant descriptions of physical states. This redundancy (usually understood as mathematical redundancy) poses challenges to identify physically meaningful quantities. Dirac (2001) argued against the possibility of measuring gauge-dependent quantities. Thus, only quantities that remain unaffected under gauge transformations are defined as observables, ensuring their physical relevance. In vacuum GR, it is difficult to construct local Dirac observables, since the gauge group of spacetime diffeomorphisms acts on spacetime points in which local objects are defined. This problem was addressed by Rovelli (2002b)), where he argued for a distinction between two notions of observability in GR. Partial observables can be observed, in the sense of measured, even if not gauge-invariant, while complete observables are constructed by relating gauge-dependent partial observables in a gauge-invariant manner and characterise the predictions of the theory. Rovelli’s proposal introduces physically meaningful observables, even if not gauge-invariant. Although theories can only predict Dirac observables, *i.e.* complete ones, gauge-dependent

[3]We consider the latter to be a problem, as gauge symmetries permeate throughout all of known physics, and it is reasonable to desire a physical interpretation for the ubiquity of gauge.

[4]We do not have space to adequately introduce this topic. For a good introduction to the distinction between active and passive diffeomorphisms see Rovelli and Gaul (2000).

partial observables are crucial in physical theories, as they describe physical observations.[5] Furthermore, introducing relational complete observables, local observability is recovered, where the locality is understood as relative localisation between fields, rather than localisation in spacetime. As we shall see, the issue of local gauge-invariant quantities in GR is strictly related to the physical significance of a reference frame.

We introduce three classes of reference frames in GR, when considered as (a set of variables representing) a material system.[6] The benefit of this constraint is due to the fact that GR is deparametrisable[7] only for some specific material models, for which it is possible to construct local Dirac observables. Thus, there is the hope that the deparametrisation techniques used for these matter toy models can provide hints to better understand the case of GR coupled with realistic standard matter.[8]

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[5]For further discussion on the concept of 'observable' in GR, see Bergmann (1961), Gryb and Thébault (2016), Pitts (2022).

[6]For simplicity, throughout the paper we will often alternate between saying that reference frames *are* physical systems and that reference systems are a set of variables *representing* physical systems. Although the difference is conceptually relevant and we think the latter is the more correct way of saying, this distinction does not invalidate our work in any way.

[7]We will directly give some examples of what deparametrising means in Section 3. Naively, it means to identify a physical degree of freedom ψ of the theory as a time. More technically, it means to rewrite the Hamiltonian constraint in the form $C = \pi + h$, where π is the conjugate momentum of ψ and where h, called the 'physical Hamiltonian', does not depend in any way on (ψ, π).

[8]However, using material reference frames is not the only option. Early attempts to use purely gravitational degrees of freedom as a reference frame in order to write local gauge-invariant Dirac observables were proposed, *e.g.*, in Komar (1958).

We call Idealised Reference Frames (**IRFs**) those reference frames that represent systems in which both the dynamical equations and the stress-energy contribution to the Einstein Field Equations (EFEs) are neglected. The second class is that of Dynamical Reference Frames (**DRFs**), whose stress-energy content is neglected, but the frame satisfies a specific dynamical equation. Finally, we name Real Reference Frames (**RRFs**) those ones whose stress-energy contribution to the EFEs is taken into account, as well as their dynamics. Although **RRFs** are systems of great interest, as they are physically more realistic in principle, in the remainder of the paper we deal exclusively with **IRFs** and **DRFs**. The proposed classification offers a possible reason why the notions of reference frame and coordinate system have not been carefully distinguished, with some notable exceptions already mentioned. The confusion stems from the practical, but not conceptual, equivalence that exists between **IRFs** and coordinate systems.[9]

Two other distinct notions are often confused, namely, 'idealisation' and 'approximation'. An idealisation is a novel, (tipically) fictitious, system that replaces the target system under study and that is simpler to analyse. Approximations are inexact descriptions of the target system. Basically, the crucial difference lies in whether one introduces a novel system (in the case of idealisation) or not (in the case of approximation). We refer the reader to Norton (2012) for details. Here, we understand **IRFs** (as well as **DRFs**) as approximations.[10] In fact, as we shall see in more detail, we proceed with approximations to a target system (represented by a **RRF**). On the other hand, coordinates are idealisations, in particular, they are mathematical objects without a physical referent (see Section 4). This paper clarifies the nature of an

[9]We mean that in theoretical practice, the use of **IRFs** or coordinates is indistinguishable. This does not mean that being aware of what one is using is irrelevant (see below).

[10]Please, forgive the unfortunate nomenclature regarding **IRFs**.

important and ubiquitous concept in physics: that of reference frame. In fact, whenever we set up an experiment or formalise the behaviour of a physical system, we implicitly or explicitly use a reference frame. The main question of the present work is what a reference frame in GR is. One reason why it is important to reach this end is that researchers in contemporary physics and philosophy of physics are interested in quantum reference frames (see Rovelli (1991a), Giacomini et al. (2019)). In fact, all physical systems are, to our knowledge, ultimately quantum. Our work can help to understand what kind of quantum reference frames are adopted when considering gravitational situations (Giacomini (2021)). We argue that, before we can really have a discussion on quantum reference frame in such framework, we should know properly what reference frames are in classical GR.

The paper is structured as follows.

In Section 2, we revise the role of reference frames and coordinate systems in gravitational physics, as well as the main definitions of a reference frame adopted in the literature

Section 3 contains the detailed classification of reference frames in GR, supported by some concrete examples

In Section 4, we provide our proposal on the origin of the lack of care in using the notions of reference frame and coordinate system in GR

In section 5 we analyse the differences between **DRFs** and coordinates. We also highlight the usefulness of reference frames to address the problem of local Dirac observables and to have a physical interpretation of diffeomorphism gauge symmetries in GR.

2 Reference Frames vs. Coordinate systems

This section is not intended to be a comprehensive review of all possible reference frame definitions. The intention is to provide a context in which to place our proposal, which extends the existing literature on the subject. We also briefly introduce the differences between coordinate systems and reference frames in gravitational and non-gravitational physics.

A local coordinate system in a N-dimensional topological manifold is unanimously considered to be a choice of a local *chart*, *i.e.* an open set and a homeomorphism γ which assign N labels to a point of the manifold. Formally, a local coordinate system (also referred to as 'coordinate chart') is defined as a 1 : 1, onto map $\gamma : \mathscr{S} \to R^N$ from an open patch $\mathscr{S} \subset M$ of the manifold M into the N-fold product of the real numbers.

Let us now summarise the main definitions of reference frame, from which all others that may be encountered are derived. In Rovelli (1991b) a reference frame is defined as a set of variables representing a material system, for example a discrete set of physical bodies or a matter field, that can be used to define a spatiotemporal localisation in a relational sense. This definition will ground our classification in Section 3. From Rovelli (1991b) also comes the suggestion that in GR reference frames can be considered 'dynamically *external*' only if they are approximated. Our proposal will extend his analysis on reference frames. On the other hand, in the work of Norton and Earman (see Earman (1974), Earman and Glymour (1978)), as is now common in the vast majority of the physical and philosophical literature of GR, a reference frame is defined by a smooth, timelike four-velocity vector field U^a tangent to the worldlines of a material system to which an equivalence class of coordinates is locally adapted.[11] It is straightforward that to fully consider the physical referentiality of this definition, the vector field, exemplified *e.g.* by massive particle worldlines, should take into account the coupled dynamics between the particle system and the gravitational degrees of freedom. On the contrary, the vector field is usually treated as not *fully* coupled with gravitational dynamics. Since this is the leading and most widely used definition, we will discuss it in a separate section (Section 3.2.3).

In the modern literature, the conventional informal way to distinguish a reference frame from a coordinate system is to point out that only a reference frame has a link to an observer's state of motion (DiSalle (2020)). Following the work of Earman and Norton, a reference frame is formally identified with a timelike four-velocity field on a manifold (see *e.g.* Bradley (2021)), or with a collection of coordinate charts on a manifold (Gomes (2023b)). Moreover, (Pooley, 2022, Sec.4.3) defines a reference frame as a set of standards (such as a standard of rest and a standard of simultaneity in Newton theory) relative to which a body's motion can be quantified. This paper demonstrates that these definitions do not fully exhaust the characterisation of reference frames in GR.

The quantum reference frame literature agrees that reference frames are associated with physical material systems, which are ultimately quantum. Relative to these objects, we determine the properties of the physical systems we wish to study, such as spatiotemporal localisation Castro-Ruiz et al. (2020). In this context, not considering reference frames as physical degrees of freedom, but as mere coordinates, has obvious consequences as one misses their quantum nature.

On the other hand, some ambiguity still appears, *e.g.* in Read (2020), in which the two

[11] Here, we are using the abstract index notation (see Penrose and Rindler (1987)) to stress that it is a geometrical object independent from the choice of a coordinate representation.

terms are not clearly differentiated. In fact, on p. 215 a (reference) frame-dependent object is defined as a 'non-tensorial object', that is thought of as a 'coordinate-dependent object' (**ivi**, p. 217). Also in Lehmkuhl (2014), following Einstein's idiom which emerges in many quotations (*ivi*, p. 321), the terms reference frame and coordinate system are used interchangeably, without much concern.

Gravitational vs. non-gravitational physics Before concluding, it is interesting to propose a comparison between the meaning of reference frames and coordinates in GR (understood as the best representative of gravitational physics), as opposed to non-gravitational theories. The need to separate the two concepts does not emerge in non-gravitational physics, since a reference frame *can* be represented by a set of degrees of freedom 'brought in from the outside' (Henneaux and Teitelboim, 1994, p.27). Outside of what? Outside the dynamical system under study. For example, within Maxwell's theory in Minkowskian spacetime, the Maxwell field is understood as a subsystem of the Universe that does not affect the global inertial reference frame that can be defined on the fixed background structure. For instance, we can define locations in spacetime by means of non electrically charged objects, which constitute the reference frame. This point is elucidated in the following passage in (Einstein, 1905, p.38):

> The theory to be developed—like every other electrodynamics—is based upon the kinematics of rigid bodies, since the assertions of any such theory concern relations between rigid bodies (systems of coordinates), clocks, and electromagnetic processes.

This passage can be interpreted to mean that the special relativistic theory is concerned with the relations between electromagnetic processes and material bodies that are dynamically *external* to (outside) the electromagnetic system under study, which Einstein calls ‘system of coordinates’.[12]

In contrast, GR has no available ‘outside’. Namely, we are not allowed to consider reference frames as dynamically *external*. This follows from the fact that no existing physical system is gravitationally neutral. Consequently, there is no way to disregard the (mutual) interaction between the gravitational field and the degrees of freedom that characterise the reference frame, unless dynamical approximations are made. But it is precisely here that the lines become blurry. In non-gravitational physics there is no need for approximations: it is always possible to define a reference frame as a ‘dynamically irrelevant’, but *real* system and to make it coincide with the notion of a coordinate system.[13] In contrast, in GR the concept of a coordinate system, even if is widely used throughout the general relativistic literature, should be considered as a mathematical artefact without a physical referent. By this, we mean that in non-gravitational physics a coordinate system *may* correspond to a system of physical objects *external* to the dynamics of the problem, while in GR a coordinate system has no a direct physical referent. If this were not the case, it would be a set of gravitationally charged degrees of freedom and thus not dynamically *external*. As we shall see, the only way to make the reference frame ‘dynamically *external*’ is to implement approximations (Section 3). So, it is clear that, conceptually, the notions of reference frame and coordinate system cannot naturally coincide in GR.

[12]For a critical analysis of this view see Pooley (2017).

[13]The reference frame might be considered *external* due to the irrelevance of interaction effects compared to the experimental precision. Nevertheless, in this context, we refer to the property of being ‘dynamically external’ independent of experimental errors and constraints.

3 IRF, DRF, RRF

Following Rovelli (1991b), we define a reference frame, at the most basic level, as a set of variables representing a material system.[14] We argue that in GR we can mean three things by the term 'reference frame'. Our novel three-fold division is based on the degree of approximation applied to the target material reference frame, which makes its dynamics more or less intertwined with that of the gravitational field. Although the paper adheres to the rationale presented in Rovelli (1991b) - that adopting approximations to define a reference frame blurs its physical significance, while considering its stress-energy presence and its dynamics brings its full physical significance back into focus - our proposal provides an independent contribution to this topic. In particular, our paper engages with more philosophical literature than Rovelli's original proposal and complements previous work with additional theoretical tools. To give a concrete example, in Rovelli (1991b) the author overlooks the class of **DRFs**, which are dealt with extensively here.[15] This fact is curious since, as we shall see, a particularly relevant physical example of **DRF** is precisely GPS coordinates, introduced in Rovelli (2002a). Moreover, to complement Rovelli's work, we place special emphasis on recognising **IRFs** as possible reference frames. In fact, we argue that they are crucial for understanding the role of coordinates in GR (section 4).

Also, this work analyses the standard definition of reference frame provided in the literature

[14]This 'level zero' definition is too broad. Conditions must be imposed on the material system. See, *e.g.*, footnote 19.

[15]In (Rovelli, 1991b, Sec. 5) he mixes and muddles between what we call **IRFs** and **DRFs**. For example, he claims to retrieve the 'local point of view' (*i.e.* the case of **IRFs**), but then adopts approximations that correspond to the case of **DRFs** (see p. 312).

in the light of a new threefold perspective and adopts a methodology that can provide a clear conceptual map useful for discerning between possible reference frames in GR. The main motivations of our proposal, which will be discussed below, concern the physical interpretation of diffeomorphism gauge freedom, as well as the need to define local Dirac observables (Section 3.2 and Section 5); the interpretation of vacuum GR; the fictitious role of coordinates in GR and the exposition of a new perspective on why the two concepts are sometimes used interchangeably without much care (Section 4).

3.1 Idealised Reference Frame

In an Idealised Reference Frame (**IRF**), the physical presence of the material system represented by the reference frame is ignored. In particular, two approximations are adopted:

(a) In the EFEs, the stress-energy tensor of the matter field used as reference frame is neglected

(b) In the system of dynamical equations, the set of equations that determine the dynamic of the matter field is neglected

Step (b) introduces some underdetermination in the gravitational dynamics, when written in terms of the matter degrees of freedom. We argue that the class of **IRFs** represents what Rovelli (2004, p.62) calls 'undetermined physical coordinates'. The reason for this designation is clearly expressed by the author:

> We obtain a system of equation for the gravitational field and other matter, expressed in terms of coordinates X^{μ} that are interpreted as the spacetime location of reference objects whose dynamics we *have chosen* to ignore. This set of

equation is underdetermined: same initial conditions can evolve into different solutions. However, the interpretation of such underdetermination is simply that we have chosen to neglect part of the equations of motion.

However, we refrain from adopting this nomenclature as it may cause unnecessary confusion between reference frames and coordinates. Basically, when we use **IRFs**, similarly to the use of coordinates in GR, the system is not deterministic. To be precise, we have not a real indeterminism, since this is the result of an unexpressed gauge freedom in the dynamics, that allows the same initial data to evolve into two different solutions. Different solutions with the same initial conditions represent two gauge-related configurations.[16] The matter degrees of freedom participate in the definition of the metric, whose evolution is determined up to four arbitrary functions because of approximation (b). The difference between **IRFs** and coordinates is very subtle and will be discussed in Section 4. We believe that the origin of the confusion between the concepts of reference frame and coordinate system stems from the pragmatic equivalence of describing physics in terms of **IRFs** or coordinates. The real difference lies only at the level of interpretation. For this reason, it is impossible to give a practical example of a physical system playing the role of an **IRF**, as it would actually coincide with a generic coordinate system.

However, as we will argue later, in order to give a physical interpretation to diffeomorphism gauge symmetries, it is crucial to acknowledge that we use approximated reference frames, not just coordinates.

[16] As for the connection between the use of coordinates in GR and indeterminism, we refer the reader to the well-known problem of the hole argument Earman and Norton (1987), Weatherall (2018), Pooley and Read (2021).

3.2 Dynamical Reference Frame

If we assume only the first of the above approximation, namely (a), we get a Dynamical Reference Frame (**DRF**). Consequently, we now have the possibility of using the dynamical equations of matter and obtain a deterministic dynamical system. The use of a **DRF** in a theory corresponds to a (completely) gauge-fixed formulation of the same theory written in coordinates. The reason is simply that we can use the equations of motion of the matter in the same way in which we use a gauge-fixing condition when we deal with coordinates. This fact supports the position expressed in Rovelli (2014) (see also (Gomes, 2023b, Sec. 2.3)), according to which the existence of a gauge is not a redundancy of the formalism, rather it suggests the relational nature of physical degrees of freedom.[17] This is also consistent with the definition given in (Henneaux and Teitelboim, 1994, p.3) of a gauge theory as a theory

> [...] in which the dynamical variables are specified with respect to a 'reference frame' whose choice is arbitrary.

According to our approach, the reference frame to which they refer is a **DRF**.

In the following, we will give three examples of a **DRF**.

3.2.1 Warm-up

Before doing so, to give a concrete and simple idea of what it means to use a **DRF**, we propose a parallel to the case of a parametrized Newtonian system in one spatial dimension, described by canonical variables $[q(t), p(t)]$. This is by no means intended to be a proper example of a

[17] In this paper, we choose to take Rovelli's position. The topic is broad and is not the focus of this work. For the sake of completeness, we refer the reader to some replies on various fronts to Rovelli's 'relational proposal' on 'why gauge'. See, *e.g.*, Teh (2015) and Weatherall (2016).

DRF, but at best a valuable analogy. Through the parametrization process we extend the configuration space $\mathscr{C} = \{q(t)\} \to \mathscr{C}_{\text{ext}} = \{q(\tau), t(\tau)\}$ and unfreeze the time coordinate t, which can now be treated as a dynamic variable on the same footing of the q variable. Both dynamical variables depend on an arbitrary parameter τ. The extended action of the parametrized system reads as

$$S_{\text{ext}} = \int d\tau \left[p_t \frac{dt}{d\tau} + p \frac{dq}{d\tau} - N(\tau) \left(p_t + \frac{p^2}{2m} \right) \right], \tag{1}$$

while the Hamilton equations are

$$\begin{cases} \dfrac{dt}{d\tau} = N(\tau), & \dfrac{dp_t}{d\tau} = 0 \\ \dfrac{dq}{d\tau} = N(\tau)\dfrac{p}{m}, & \dfrac{dp}{d\tau} = 0 \end{cases} \tag{2}$$

The extended system is subject to the reparametrization symmetry $\tau \to \tau'(\tau)$ and different choices of the Lagrange multiplier $N(\tau)$, also known as ‘lapse function’, amount to considering the gauge dynamics in different parametrizations τ. We partially gauge fix the system, through the gauge choice $N = 1$, which amounts to having a parametrization in which $t(\tau)$ grows linearly. However, the dynamics is still a gauge dynamic and not a physical one, since it is expressed in terms of a non-physical parameter. Thus, we still have a gauge redundancy in the system and we cannot define gauge invariant quantities representing Dirac observables.

A well-known approach to constructing local gauge-invariant observables is to impose the canonical gauge condition $t(\tau) \equiv t_0$, which completely eliminates any residual gauge redundancy. Geometrically, this condition defines a slice that cuts all the gauge orbits on the constraint surface once and only once. That way, we can write observables which are understood as gauge-invariant extensions of a gauge-fixed quantity. In particular, an observable can be defined as the coincidence of q with t, when t reads the value t_0, or explicitly: $q(\tau)|_{t(\tau)=t_0} := q(\tau) + p/m\,[t_0 - t(\tau)]$. Note that this is the definition of an evolving constant of motion (see Belot and Earman (2001)).

An equivalent approach to pick the variable t as the 'temporal reference frame' (also referred to as the internal, or relational 'clock') is to simply invert the relation $t(\tau) = \tau \leftrightarrow \tau(t) = t$. Note that this can be done since we are able to solve Hamilton equations for the considered system. Furthermore, in this case, the function $t(\tau)$ is globally invertible, but this is not always the case. By inserting the quantity $\tau(t)$ within the gauge-dependent quantity $q(\tau)$ we obtain a gauge-invariant relational observable $q(t)$, defined for any given value of t, thus deparametrising the system.[18] Consequently, we recover the formalism of the unparametrized case in which t represented a mere coordinate. However, in such a case the physical interpretation of the time t as a dynamical variable is now revealed and, in this sense, it represents a good analogy of a component of a **DRF**. In fact, now $q(t)$ describes the gauge-invariant relational evolution of q with respect to the dynamical variable t. Furthermore, the dynamical theory written in relational terms becomes deterministic and without any gauge redundancy.

3.2.2 Four Klein-Gordon scalar fields

The first proper example of a **DRF** is the so-called *test fluid* reference frame (see Wald (1984)). In short, the test fluid is affected by the metric field (it is acted upon), but the metric field is not

[18] Apart from technical details, this is how complete observables are constructed (see Dittrich (2006), Tambornino (2012)).

affected by the test fluid (it does not act): thus, the back-reaction on gravity (namely, its stress-energy tensor on the *r.h.s.* of the EFEs) is approximated to be negligible. As a toy model for a test fluid, we consider a set of four real, massless, free, Klein-Gordon scalar fields in a curved spacetime. Each scalar field $\phi_{(A)}, A = 1,2,3,4$ has its own equations of motion (in abstract index notation)

$$\Box\phi_{(A)} \equiv \nabla^a \nabla_a \phi_{(A)} = 0 \tag{3}$$

and the system of the four scalar fields can be used as a reference frame (a clock and three rods) with respect to which local observables can be defined.[19] More clearly, to describe the dynamics of the scalar fields we first need to know the metric g_{ab} of spacetime in order to define the compatible connection ∇_a. In that sense, the metric acts on the test fluid, but it is not affected by it. This example clarifies what we have said about the correspondence between using a **DRF** and a gauge-fixed formulation of the theory written in coordinates. In fact, the four scalar fields satisfy the same equation that is written when De Donder gauge-fixing is imposed on coordinates. Hence, we have a straightforward example of a gauge-fixing, understood relationally.

When we use the set of Klein-Gordon fields $\phi^{(A)}$ as the reference frame, we recover local gauge-invariant observables, *e.g*, $g_{AB}[\phi^C]$. No gauge redundancy appears. With this notation, it is emphasised that the metric is not defined on the space-time manifold $\mathscr{M}$, but on a somewhat 'physical' manifold $\mathscr{T}$, which is the space of fields' values. In this sense, reference fields $\phi^{(A)}$

[19]Arguably, the scalar field selected to play the role of the timelike variable (say $\phi^{(1)}$) needs to satisfy some properties such as a homogeneity condition $\nabla^i \nabla_i \phi^{(1)}(x^\mu) = 0$, where $i = 1,2,3$ are spatial indices in some coordinates $\{x^\mu\}$. We could also assume a 'monotonicity condition' connected with some assumptions on its potential (when it is considered).

can be understood as diffeomorphisms $\phi^{(A)} : g_{ab} \in \mathscr{M} \to g_{AB} \in \mathscr{T}$.[20] Taking away any reference to spacetime manifold points, one obtains a well-defined notion of local gauge-invariant observables in GR and a definition of a physical spacetime point in terms of 'Einsteinian coincidences' (Einstein (1916)). Physical objects do not localise relative to the manifold, but relative to one another. This constitutes what is referred to as *relational localisation* (see Goeller et al. (2022)). Basically, as we have shown, one must express the spatiotemporal localisation of observables through matter fields, which play the role of reference frames. Thus, we designate all the spatiotemporal locations by the values of the scalar fields. As also argued in Gomes (2023a), this way of understanding localisation in terms of physical field values is very similar to Einstein's original understanding of coordinates (see footnote 1 in the Introduction).

3.2.3 DRFs in the orthodox view

In accordance with the previously mentioned literature (see Section 2), a **DRF** could also be represented by a timelike four-velocity vector field U^a tangent to a congruence of worldlines of a particle system, or a matter fluid. We choose to analyse this particular case in a depth, as we believe it warrants a closer examination and it is particularly significant from an historical and philosophical perspective.

The 'orthodox' point of view (that is how we call the view introduced in the philosophical

[20] In some coordinates $\{x^\rho\}$, we have explicitly $g^{\alpha\beta}[\Phi^\gamma] = (\partial_\mu \Phi^\alpha)(\partial_\nu \Phi^\beta) g^{\mu\nu}(x^\rho)$. With a slight abuse of notation we have defined $\Phi^\gamma(x^\rho) := \{\phi^0(x), \phi^i(x)\}, i = (1,2,3)$. See (Goeller et al., 2022, p.18). The initial letters $(\alpha, \beta, ...)$ of the Greek alphabet refer to reference frame's indices. The final letters $(\mu, \nu, ...)$ refer to coordinates' indices.

literature by Earman and Norton)[21] recognises as the only viable characterisation of a reference frame the expression of matter's state of motion, *i.e.* the assignment of a four-velocity (timelike) vector field U^a tangent to the worldlines of a material system, satisfying some dynamical equation and to which locally adapt a coordinate system (x^0, x^1, x^2, x^3). A straightforward example of such a reference frame is a fluid of matter falling (not necessarily freely) towards, *e.g.*, a black hole.[22] Physical space is the set of points of the fluid, *viz.* the hypersurface orthogonal to the four-velocity, while the degree of freedom playing the role of time is a scalar quantity which grows monotonically along the fluid trajectory. In this case, the fluid and its physical ingredients, like energy density or pressure satisfy a precise prescription for their dynamical evolution, but since such matter typically produces only a small perturbation on the spacetime structure of the black hole, its physical back-reaction on spacetime is neglected.

To complete the picture, we quote the definitions of a reference frame provided by Earman and Norton.

> In this context a reference frame is defined by a smooth, timelike vector field V. [...]Alternatively, a frame can, at least locally, be construed as an equivalence class of coordinate systems. The coordinate system $\{x^i\}$, $i = 1,2,3,4$, is said to be adapted to the frame F if the trajectories of the vector field which defines F have the form $x^a = \text{const}, a = 1,2,3$. If $\{x^i\}$ is adapted to F, then so is $\{x'^i\}$ where $x'^a = x'^a(x^b)$, $x'^4 = x'^4(x^b, x^4)$; such a transformation is called an internal

[21] It is also the most supported in the physical literature (see Wald (1984), Malament (2012)).

[22] Arguably, this example loses its validity near the singularity, where quantum effects become dominant. Furthermore, at the singularity there is no longer a congruence of worldlines, as the geodesics intersect.

coordinate transformation. F may be identified with a maximal class of internally related class of coordinate systems. (Earman, 1974, p.270)

[...] it is now customary to represent the intuitive notion of a physical frame of reference as a congruence of time-like curves. Each curve represents the world line of a reference point of the frame. [...] A coordinate system $\{x^i\}$ $(i = 1,2,3,4)$ is said to be 'adapted' to a given frame of reference just in case the curves of constant x^1, x^2 and x^3 are the curves of the frame. These three coordinates are 'spatial' coordinates and the x^4 coordinate a 'time' coordinate. (Norton, 1985, p.209)

Of course, since we defined a **DRF** as a physical material system that satisfies equations of motion and depends on, but not affects, the gravitational field, we can firmly assert that the orthodox view considers **DRFs** as possible reference frames. However, there is no mention of reference frames corresponding to **IRFs**, or **RRFs**. The proposed comparison between reference frame 'à la Earman-Norton' and **DRFs** can be useful both for a better understanding of **DRFs**, but above all for providing a more delimited conceptual context to reference frames as usually used in the literature.

We would like to comment briefly that there are possible differences, to be analysed, between a **DRF** and a reference frame in the orthodox sense. However, we will not engage in this project on this occasion. We only mention that, contrary to the given definition of a **DRF**, the orthodox characterisation of a reference frame as a maximal class of adapted coordinate systems can lead to conceptual confusion between the set of adapted coordinates and the set of dynamical degrees of freedom constituting the reference frame. This is also stated in (Earman and Glymour, 1978, p.254):

Of course, a reference frame can be represented by a maximal class of adapted coordinate systems. [...] *But such a coordinate representation can easily lead to a blurring of the crucial distinctions* [between reference frames and coordinate systems] *mentioned above*. (Our italics).

3.2.4 GPS coordinates

To conclude, we argue that a realistic example of **DRF** is represented by the set of the so-called *GPS coordinates*, introduced in Rovelli (2002a). The idea is to consider the system formed by GR coupled with four test bodies, referred to as 'satellites', which are deemed point particles following timelike geodesics. Each particle is associated with its own proper time τ. Accordingly, we can uniquely associate four numbers $\tau^{(A)}, A = 1,2,3,4$ to each spacetime point P. These four numbers represent the four physical variables that constitute the **DRF**. This example in particular highlights the usefulness of our classification as a clear and easy-to-use conceptual framework for categorising reference frames introduced in the literature.

3.3 Real Reference Frame

When we take into account both the dynamics of the chosen material reference frame and its stress-energy tensor, we get a Real Reference Frame (**RRF**). Examples of **RRFs** include pressureless dust fields Brown and Kuchař (1995) and massless scalar fields Rovelli and Smolin (1994). It is worth noting that such reference frames are considered for reasons of mathematical convenience, rather than for their clear phenomenology. Moreover, while they lead to a complete deparameterisation of the theory, this is not always the case for other **RRFs**. In fact, it is possible to propose a sub-classification of **RRFs** based on the possibility of being able to deparametrise the theory. As a matter of fact, in some cases approximations can be made to the Hamiltonian of the material field used as the **RRF**, thereby implementing a deparametrisation procedure.[23] However, when no such approximations can be made, the resulting approach, while physically more stringent (or real), is formally more complicated and does not allow for the analytical control of relational observables Tambornino (2012). In the remainder of the paper we will primarily focus on **IRFs** and **DRFs**, leaving the study of **RRFs** for future research.

Summing up The relevance of introducing our classification can be summarised as follows (see also Section 5):

- **Conceptual Framework**: it provides a clear conceptual framework for discussing issues in GR related to reference frames as material systems coupled to gravity. In particular, it helps contextualising the issues presented in the Introduction and distinguishing the implications of using different types of reference frames:

  (P1) We cannot define local Dirac observables when we use **IRFs**, or coordinates

  (P2) The gauge freedom of GR is interpreted as a mere mathematical redundancy if we use **IRFs**, or coordinates.

  However, using reference frames and by relaxing some of the approximations, thus considering **DRFs**, or **RRFs**, both problems (P1) and (P2) find a natural resolution (see

[23]Therefore, even in the case of **RRFs** we have room to make some approximations in the sense of Norton (2012).

Section 5).[24] To be clear: our proposal is not intended to be a new proposal to solve these problems. It is not. It provides a new theoretical structure in which to frame such problems.

- **Effective Communication**: it provides a valuable semantic clarification, thus enabling clear communication on the use of reference frames in GR
- **Enhanced Understanding**: it enhances understanding on the role of material reference frames in GR, confirming their practical significance to resolve the identified issues. In addition, it also helps to understand the relationship between coordinates and reference frames in GR (see next Section 4).

We conclude by considering another interesting implication in considering **IRFs** and **DRFs** as possible classes of reference frames. If we disregard also any stress-energy contribution from other material sources, the solutions of the EFEs will be vacuum solutions. In support of Rovelli (1991b), we can say that vacuum GR can be seen as an approximate theory, in which we make use of **IRFs**, or **DRFs**. In other words, we are suggesting that *exact* vacuum solutions may not exist in nature, but only *approximated* matter solutions that behave like vacuum solutions could be permitted.

[24] We recall that our approach is not restricted to a coordinate formulation of a theory. Gauge freedom under active diffeomorphisms appears as a mathematical redundancy because one adopts spatiotemporal localisation in terms of manifold points. Once one adopts relational localisation and replaces the manifold points with the components of a reference field, one obtains gauge-invariant observables and it becomes clear why gauge existed.

4 IRFs vs. Coordinates: what is the source of the confusion between reference frames and coordinate systems?

From the theoretical point of view, in GR local coordinate systems are employed to compute solutions of EFEs. A straightforward example is the use of Schwarzschild coordinates (t, r, θ, ϕ). Of course, the Schwarzschild geometry can be expressed in a range of different choices of coordinates. As far as experimental practice is concerned, a notable empirical success of GR is the detection of gravitational waves by the LIGO project (Abbott et al. (2016)). The gravitational contribution of mirrors used to detect gravitational waves on Earth is completely disregarded and their degrees of freedom are treated as coordinates. Even theoretically, the components of the metric are calculated within a particular gauge, namely the so-called Transverse-Traceless gauge (TT gauge).

Therefore, in GR a reference frame is often used as a coordinate system. We mean that, strictly speaking, from our point of view also Schwarzschild coordinates (t, r, θ, ϕ) should represent some material degrees of freedom. However, they are idealised (in the sense of Norton (2012)) to coordinates, without *any* reference to a material system substantiating them. The implications of this statement will require further investigation in the future. As already mentioned at the end of Section 3, adopting such an approach could have significant consequences for our understanding of vacuum solutions in GR.

Similarly, the components of the metric predicted and then measured by LIGO should not be interpreted as quantities written in coordinates, but as written in some reference frame that represents the mirrors of the experimental setting to some degree of approximation. This is because we know that in order to define local observables (in the Dirac sense), the location must be relative between fields and not relative to manifold points to which we assign coordinates. The main message is: the most we can do is to approximate (in the sense discussed in this paper) the physical systems we choose as reference frames. Thus, *at most*, we can have **IRFs** that behave exactly like coordinates, but do not coincide with coordinates. In GR, what we call coordinates should instead be interpreted *at most* as variables of an **IRF**, but the two are confused, since pragmatically identical. However, acknowledging that we are using reference frames can help us understand the physical reasons for presence of diffeomorphism gauge freedom. Namely, that the presence of diffeomorphisms as gauge redundancy indicates the tacit assumption of an approximation procedure that eliminates the dynamic presence of a physical system we are using as a reference frame.

The puzzle, then, is why such approximations work so well and, consequently, why the idealisation of reference frames as mere coordinates works so well that the difference between the two concepts can be overlooked. This issue is clearly expressed by Thiemann (2006, p.2):[25]

> Why is it that the FRW equations describe the physical time evolution which is actually observed for instance through red shift experiments, of physical, that is observable, quantities such as the scale parameter? The puzzle here is that these observed quantities are mathematically described by functions on the phase space which *do not Poisson commute with the constraints*! Hence they are not gauge invariant and therefore should not be observable in obvious contradiction to reality.

Simply put, in theoretical and experimental practice reference frames are approximated to **IRFs**, but understood as coordinate systems. At the practical level, it is impossible to separate between the two concepts. In both cases, there is gauge-redundancy and no local Dirac observables can be defined. However, this leads to the situation where all general-relativistic

[25] Thiemann is referring here to Dirac observables.

physics incorrectly interprets the dynamical equations of systems as physical evolution equations 'rather than what they really are, namely gauge transformation equations' (*ivi*, p.3), as they are written in coordinates. The analysis of such a puzzle deserves a separate discussion, which will not be carried out here. Let us just say that our classification serves us well. In fact, only by acknowledging that we are using **IRFs**, and not coordinates, we can lighten the degree of approximation on the target system, thus including the dynamical equations. In this way, the dynamics becomes a physical dynamics of local gauge-invariant observables written in terms of **DRFs** and not a (coordinate) gauge-dependent description of reality.

According to us, the underlying source of the confusion between coordinate systems and reference frames is that reference frames are approximated to such an extent that they play the role of **IRFs**. However, once these approximations are made it becomes impossible to realise that approximated physical systems in the sense of **IRFs**, rather than coordinate systems, are being used. The relevant point is that *in practice* there is no difference between a coordinate system and an **IRF**. Both come in the form of a set of non-dynamic variables that are used to define a spatiotemporal location. The difference between **IRFs** and coordinate systems all plays out on the conceptual level. An **IRF** represents a physical systems that would, by nature, interact with all other degrees of freedom in the theory, but to which we apply *a posteriori* various approximations. On the other hand, a coordinate system is an idealisation: it is a set of mathematical variables that *naturally* have no dynamics whatsoever (that's an *exact* description of the system). In a nutshell: coordinates are mathematical idealisations without a physical referent; **IRFs** are approximations of a target physical system. Hence, the confusion between coordinates and reference frames can be traced back to that between idealisations and approximations, and 'the difference matters' (Norton (2012)).

From a more technical point of view, the distinction between **IRFs** and coordinates is rooted in the fact that coordinates are not partial observables, whereas the variables that constitute an **IRF** are and can be associated to measurements performed by instruments.[26]

5 DRF vs. Coordinate systems

The differences between **DRFs** and coordinates are clear and can be summarised as follows:

- The gravitational dynamics is deterministic when using a **DRF** and not deterministic (in the sense of gauge-freedom) when using coordinates
- We can define local Dirac observables in terms of a **DRF**. No local Dirac observables are defined when we use coordinates
- The variables constituting a **DRF** are partial observables, while coordinates are not

Let us give a practical example of such differences. Let $\{x^0, x^i\}$ be a set of coordinates and $\{T, Z^i\}$ a set of four scalar degrees of freedom, defined by some dynamical equations. We consider the ADM space+time analysis of a model of GR plus a pressureless dust fluid described by the set $\{T, Z^i\}$, playing the role of a **DRF**.[27] In total we have $6 \times \infty^3$ degrees of freedom of the 3-metric $h_{ij}(x^0, x^i)$ written in coordinates and $4 \times \infty^3$ physical degrees of freedom of the material system. By removing the $4 \times \infty^3$ gauge degrees of freedom of the metric, $6 \times \infty^3$ physical degrees of freedom remain. When we use the dust fluid as a reference frame, the metric $h_{\alpha\beta}(T, Z^i)$ has $6 \times \infty^3$ physical degrees of freedom.[28] Thus, the same

[26]It should be noted that the quantities that define a **DRF** are also partial observables. However, in the case of partial observables forming an **IRF**, we neglect their dynamics.

[27]The ADM formalism Arnowitt et al. (1960) is a Hamiltonian formulation of GR. The canonical variables of this formalism are the 3-metric tensor h_{ij} and its conjugate momentun p^{ij}.

[28]For the index notation used here, see footnote 20.

dynamical theory, when written in relational terms (*i.e.* using a **DRF**) is well-defined without any gauge freedom to be fixed. Furthermore, the (relationally) local quantity $h_{\alpha\beta}(T, Z^i)$ commutes with all constraints of the theory, hence it is a local Dirac observable. In fact, the diffeomorphism group act both on the 3-metric and the scalar fields, in such a way to leave $h_{\alpha\beta}(T, Z^i)$ invariant.

This example shows that the use of **DRFs** solves both problems (P1) and (P2).

(P1-solved) As also demonstrated with practical examples in section 3.2, we are able to write local Dirac observables. The price to pay is to accept a notion of relational locality between fields. The force of this notion lies in the possibility of constructing gauge-invariant local observables and defining a physical dynamics for such quantities.

(P2-solved) As far as the interpretation of diffeomorphism gauge freedom in GR, using **DRFs**, we can interpret gauge-fixing conditions as dynamical equations of some physical system chosen as the reference frame. The presence of gauge freedom, in such a view, suggests that we are ignoring the dynamical presence of some physical system that we are using as a reference frame. In other words, as stated in Rovelli (2014):

> Gauge invariance is not just mathematical redundancy; it is an indication of the relational character of fundamental observables in physics. [...] Gauge is ubiquitous. It is not unphysical redundancy of our mathematics. It reveals the relational structure of our world.
>
> [...] The choice of a particular gauge can be realized *physically* via coupling: with a material reference system in general relativity.

6 Conclusion

The presented work introduced three distinct classes of reference frames in GR, according to their increasing physical role in the gravitational dynamics. Indeed, we considered 'idealised' (**IRF**) those reference frames whose physical nature does not enter in any way into the physical picture, as 'dynamical' (**DRF**) those one which are associated with a specific set of dynamical equations, as 'real' (**RRF**) those whose stress-energy tensor also contributes to the EFEs. In light of the identified problems related to defining local Dirac observables and interpreting diffeomorphism gauge not as mere mathematical redundancies, our novel three-fold classification proved to be a valuable tool, providing a theoretical framework in which to effectively contextualise the aforementioned challenges. In particular, we analysed the role of **DRFs** in this debate. This work complements and extends existing literature on the subject, as it includes past definitions of a reference frame, thus enhancing the coherence and validity of the proposed framework.

The recognition of the potential risk in confusing reference frames with coordinates sheds light on the necessity of maintaining conceptual clarity to avoid significant errors in interpretation. Reference frames can be approximated as **IRFs** and inaccurately conflated with coordinate systems, since an **IRF** behaves *as if* it were a coordinate system. On a conceptual level it is a serious mistake to confuse the two notions. Coordinates are mathematical idealisations without a physical referent. **IRFs** are approximations of a target physical system.

Our proposal on reference frames could have implications both for the increasingly studied notion of quantum reference frame and for future discussions on the nature of vacuum solutions of EFEs. In particular, it remains to be clarified if vacuum solutions can be reconsidered in terms of approximated matter solutions where the stress-energy tensor is neglected. This is not the only option. Certainly, it is possible to opt for a definition of a non-material reference frame in vacuum GR, for example in terms of purely gravitational degrees of freedom. Likewise, it remains to be clarified why and to what extent the idealisation of reference frames as mere coordinates works so well that the difference between the two concepts can be overlooked. This observation prompts us to consider why equations written in coordinates still yield predictive results, although theoretically, physical predictions should solely arise from equations expressed in terms of reference frames. The measurement of gravitational waves, which is one of the greatest experimental successes in GR, is the clearest example of such a conundrum.

The proposed framework serves as a valuable guide for researchers, offering new perspectives, confirming some well-established research paths and opening avenues for exploration within the field. Overall, this work enriches the study of a sector of GR, by providing a clear and coherent approach to understanding the role of reference frames as material systems coupled to gravity. In conclusion, our systematic classification of reference frames may have significant implications for the foundations of Einsteinian theory of gravity.

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