The role of idealizations in the Aharonov-Bohm effect

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Abstract On standard accounts of scientific theorizing, the role of idealizations is to facilitate the analysis of some real world system by employing a simplified representation of the target system, raising the obvious worry about how reliable knowledge can be obtained from inaccurate descriptions. The idealizations involved in the Aharonov-Bohm (AB) effect do not, it is claimed, fit this paradigm; rather the target system is a fictional system characterized by features that, though physically possible, are not realized in the actual world. The point of studying such a fictional system is to understand the foundations of quantum mechanics and how its predictions depart from those of classical mechanics. The original worry about the use of idealizations is replaced by a new one; namely, how can actual world experiments serve to confirm the AB effect if it concerns the behavior of a fictional system? Struggle with this issue helps to account for the fact that almost three decades elapsed before a consensus emerged that the predicted AB effect had received solid experimental support. Standard accounts of idealizations tout the role they play in making tractable the analysis of the target system; by contrast, the idealizations involved in the AB effect make its analysis both conceptually and mathematically challenging. The idealizations required for the AB effect are also responsible for the existence of unitarily inequivalent representations of the canonical commutation relations and of the current algebra, representations which an observer confined to the electron's configuration space could invoke to 'explain' AB-type effect without the need to posit a hidden magnetic field. The goal of this paper is to bring to the attention of the philosophers of science these and other aspects of the AB effect which are neglected or inadequately treated in literature.

Keywords Idealizations · Aharonov-Bohm effect· Foundations of quantum mechanics · Non-locality · Unitarily inequivalent representations

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1 Introduction

In recent years philosophers of science have devoted an increasing amount of attention to the role that approximations, abstractions, and idealizations play in scientific theorizing; but in view of the complexities involved it is not surprising that no consensus account has emerged for how these notions are interrelated and how they function in scientific theorizing.[1] It seems fair to say, however, that almost all of the attention in the literature on idealizations in physics has focused on one sense of idealization: the target system is an actual system, which may be as simple as a hydrogen atom or as complex as the earth's climate system or even the entire cosmos; the purpose of the idealization is to further understanding of the hows and why of the behavior of the target system and/or to facilitate predictions about some aspect of its behavior; and the idealization involves the intentional use of falsehoods or distortions in representing (or modeling, if you prefer) the target system. This sense of idealization gives rise to an obvious puzzle; namely, how can representations involving falsehoods and distortions function to deliver scientific knowledge and explanation about the target system? And if not knowledge and explanation, what do they deliver? Again it is not surprising that there is no consensus answer to these questions or to the related question of whether idealizations are essential to or ineliminable from theorizing in physics.

No attempt is made here to contribute to the discussion of these issues. Rather the focus is on a different sense of idealization, rarely identified as such in the philosophical literature: here the target system is an idealization in the sense of a fictional system, a system which is compatible with what in the context of inquiry is taken to be a fundamental theory of physics, but which is not realized in the actual world. On the first sense of idealization, where the target system is an actual world system, an effect is dismissed as a mere artifact of the idealization if it disappears when the idealization is made more realistic. On the second sense of idealization the center of interest is on effects that should, according to said theory, be exhibited by the fictional system. The goal of studying such effects is to illuminate the foundations of said theory and its relationship to predecessor and to competing theories. The puzzle associated with the first sense of idealization does not apply here

[1] Two recent and admirably clear, but contrasting, accounts of idealizations can be found in Norton (2012) and Weisberg (2007, 2013).

since a false or distorted representation of the target system is not involved; rather one operates with a precise and exact description of the target system, which is by construction the fictional system satisfying said description. But in the place of the first puzzle another arises; namely, how can experiments on actual systems serve to confirm the predictions of the theory for the behavior of the fictional system?

The claim made here is that the Aharonov-Bohm (AB) effect is best viewed as a case of an idealization in the second sense and, further, that part of the controversy that has swirled around the experimental testing of this effect is a case of the second puzzle in action.[2] Admittedly, when one first reads a description of a system that illustrates the AB effect involving, for example, an infinitely long solenoid that perfectly contains its magnetic flux, the word that pops to mind is "idealization," and since the first sense of idealization outlined above is so dominant one naturally supposes that said solenoid is being used to model an actual world system. Only a little further reading of the physics literature is needed to dispel this misimpression. What draws the interest of physicists and philosophers alike about the AB effect is the (alleged) prediction of QM that the behavior of an electron depends on the value of the magnetic flux in circumstances where the electron never enters a region where the magnetic field is non-zero. But such circumstances, although compatible with QM, are never realized in the actual world. Thus, the target system in the AB effect is a fictional system, and there is no idealization in the usual sense—no distorted/false description of an actual world arrangement of magnets and electrons—but rather an accurate and precise description of an other-worldly arrangement.[3] The AB effect also satisfies in spades the other characteristic of the second sense of idealization, for from its discovery down to the present day the AB effect has been the locus of a lively discussion of the foundations of QM.

To avoid needless confusions it would, perhaps, be best to drop the word "idealization" in discussing the AB effect. But whatever labels are used the discussion must confront the question of how experiments on actual magnets

[2] A classic presentation of the theoretical and experimental aspects of the AB effect is to be found in Peskin and Tonomura (1989). A more up to date summary of experimental results and applications is to be found in Tonomura (2010). For a sampling of the philosophical literature on the AB effect, see Batterman (2003), Healey (1997, 2007), Lyre (2001, 2009), Mattingly (2006), Maudlin (1998), Nounou (2003), and Wallace (2014).

[3] Approximations are used in deriving a quantitative expression for the AB phase shift; but these approximations are under good mathematical control (see Section 5).

and electrons serve to confirm predictions of QM about what would be measured in a fictional arrangement required for the AB effect. Of course, there is a sense in which there is no general problem here since the confirmation of predictions of a theory about actual world circumstances lends indirect support to all of the theory's predictions. But initially the AB effect seemed so counterintuitive that some physicists doubted that it is a valid prediction of QM, and in such circumstances it was natural that more direct confirmation was demanded. It might seem that the response to the demand is simple; namely, confirmation of the AB effect is obtained when actual laboratory conditions are sufficiently close to those that characterize a fictional AB system. As will be seen, however, there were critics who argued, in effect, that close is never close enough. For what is disturbing about the AB effect is that the magnetic field seems to act where it is not; but, the critics argued, for any real world experiment they could provide an explanation of the observed effects entirely in terms of the interaction of the electron with the magnetic field where it is non-zero. Such skepticism could be countered by showing that a fictional AB system can be obtained as a well-defined limit of a series of actual world systems. Thus, once actual world experiments confirm predictions for systems sufficiently far along in the series any remaining skepticism about predictions in the limit degenerate into vulgar inductive skepticism. Such limit results will be discussed in what follows. But at the same time it should be realized that the skeptics can have a last laugh since in the limit there emerge unitarily inequivalent representations of the canonical commutation relations (CCR), and the AB effect can be attributed to a choice of representation inquivalent to the familiar Schrödinger representation rather than to a magnetic flux that exists outside the electron's configuration space. But this is getting far ahead of the story.

To return to situating this paper in the context of the literature on idealizations, the second sense of idealization involved in the AB effect does not fit happily in the most cited attempts to taxonomize idealizations (e.g. Weisberg's 2007, 2013).[4] More generally, only recently have philosophers begun to pay attention to the use of idealizations in exploring the foundations of physical theories, and much remains to be done.[5] Also missing from

[4]This point is argued in detail in Shech and Gelfert (2016).

[5]The most explicit and detailed account of the exploratory role of idealizations is to be found in Shech and Gelfert (2016); see also Shech (2015, Section 7), who also discusses methodological and pedagogical roles of idealizations. But philosophers have not been completely deaf to such themes; see Redhead (1980) and Yi (2002).

the philosophical literature is any sense of just how subtle, complex, and rich an analysis is required for the fictional systems that display the AB effect.[6] Such subtlety and complexity are in contrast with what (allegedly) happens with one species of the first sense of idealization—so-called Galilean idealizations—where the falsehoods and distortions of the representation are used to simplify and render computationally tractable the treatment of the target system (see McMullin 1985).[7] In what follows I attempt to provide an overview of some of the methodological and foundational issues that have escaped attention in the philosophical literature, and I will offer some heterodox opinions on the issues that have attracted the most attention in the literature.

The plan of the paper is as follows. Section 2 introduces one set of idealizations used in typical presentations of the magnetic AB effect—an infinitely long solenoid with various other fictional attributes. Section 3 reviews some mathematical background needed for the analysis of the AB effect, and these tools are used to discuss one of the early attempts to dismiss the effect as a mathematical artifact. Section 4 reveals how the idealizations underlying the AB effect lead to a potential stumbling block to deriving from the Schrödinger dynamics of the electron a quantitative prediction of the interference pattern exhibited by beams of electrons passing by opposite sides of the idealized solenoid; namely, the Hamiltonian operator for the electron is not essentially self-adjoint, and until further considerations are brought to bear there is no unambiguous Schrödinger dynamics. An attempt to justify a particular self-adjoint extension is discussed. Part of the justification is supplied by a pretty mathematical result, but also needed is a fleshing out of the idealization with additional stories about how the fictional attributes are realized. Section 5 analyzes key assumptions that go into standard derivations of the phase shift in the magnetic AB effect. The derivation can be seen as an attempt to apply the apparatus of QM to answer a counterfactual question: What would happen under idealized conditions to the electron interference pattern when the solenoid is switched on and off? The answer obtained apparently does not depend on the choice of a particular self-adjoint extension of the Hamiltonian operator, at least if the boundary conditions at the solenoid border are the same in the 'on' and 'off' versions of the counterfactual scenarios. While this

[6] Shech (2015) and Shech (2017) are notable exceptions.

[7] However, simplification and computational tractability are most certainly not features of some of the most discussed cases of idealization, such as the thermodynamic limit in models of phase transitions; see Ruetsche (2011, pp. 284-287).

is a reasonable assumption, there is nothing in the quantum theory itself to justify it, and only an appeal to additional stories about the realization of idealized features is availing. In any case, it is pointed out that the AB effect in a broad sense should read AB effect*s* (plural), involving more than the phase shifts that occupy most of the attention in the literature. These other effects most definitely do depend on the choice of self-adjoint extension of the Hamiltonian operator. Section 6 switches from the God's eye perspective in which all of physical space can be observed to that of an embodied observer who can make measurements only within the electron's configuration space. The AB-type effects experienced by such an observer will be attributed by a scientific realist to the presence of a magnetic field that is not accessible to our observer since it lies outside her configuration space. But an alternative explanation, not invoking unobservable magnetic fields or other hidden causes, is available due to the fact that our observer has available a choice among unitarily inequivalent representations of the CCR. In this way issues—all too familiar to philosophers of science—about realist vs. instrumentalist interpretations of scientific theories find an interesting application in the analysis of the AB effect. Section 7 discusses experimental tests of the AB effect and, in particular, why confirmation of the effect was initially so controversial and how the controversy eventually subsided. Section 8 takes up critical reaction to Aharonov and Bohm's suggestion that the AB effect demonstrates that electromagnetic potentials have a physical significance in quantum electrodynamics that they lack in classical electrodynamics, and it traces how this issue was transmuted into the issue of nonlocality of quantum observables. Conclusions are given in Section 9.

2 The AB effect in outline

2.1 Historical note

The AB effect is so-called because of the seminal influence of the Aharonov and Bohm (1959) paper—the *Physical Review* counts 3,364 citations. But as is often the case with scientific discoveries, this one was anticipated by other researchers.[8] The most important precursor is to be found in the paper of Ehrenberg and Siday (1949) in which what is now called the mag-

[8] The most complete extant account of the pre-history of the AB effect is to be found in Hiley (2013).

netic AB effect is formulated, leading some commentators to propose that a proper moniker would be the ESAB effect (see Strocchi and Wightman 1974). Sturrock and Groves (2010) proposed using the "Ehrenberg-Siday effect" to refer to what is commonly called the magnetic AB effect while reserving the "Aharonov-Bohm effect" for the electric effect. But others feel that the standard moniker is appropriate (see Berry 2010).

Reportedly, Ehrenberg and Siday initially thought that their derivation must contain some flaw, and they sought the help of colleagues in ferreting it out. Siday's exasperation in not getting useful advice is recorded in his remark: "There are all these sodding geniuses poncing around but if they ever have to do anything – oh yes, that is a different matter isn't it!" (quoted in Hiley 2013, p. 10). When Siday met with Max Born to explain the issue, reportedly Born left the meeting "with a face looking like thunder" (ibid.). Despite some misgivings, Ehrenberg and Siday eventually decided to publish, but their paper did not draw much attention, both because it was directed at the relatively small audience of researchers in electron optics, and because they did not highlight the implications of the effect for the foundations of QM. The Aharonov and Bohm paper by contrast put these implications front and center, and it drew immediate widespread attention, a good bit of which was initially skeptical. The skepticism was directed both to the genuineness of the effect itself (see Section 3.3) and to Aharanov and Bohm's suggestion that the electromagnetic potentials, rather than the fields, play a fundamental role in the laws of quantum physics (see Section 8).

2.2 The magnetic AB effect

A typical presentation of the AB effect employs a fictional system with the following features:

(F1) An infinitely long cylindrical shaped solenoid $\mathcal{S}_\infty$.

(F2) When the current is turned on in the solenoid the magnetic field $\boldsymbol{B}_\infty$ generated is completely contained within the solenoid.

(F3) The solenoid is impenetrable to an external electron.

Needless to say, none of these features is realized in nature: actual solenoids can be long but they do not stretch to infinity; actual solenoids leak flux;

and potential barriers may be high but not infinitely high.[9]

This fictional system is embedded in a more realistic two slit apparatus, as shown schematically in Fig. 1. A beam of electrons emitted from the source, and as it moves through the apparatus it is split in two parts that pass $\mathcal{S}_\infty$ on opposite sides. When the beams are recombined at the screen, an interference pattern is exhibited. According to the Aharonov-Bohm analysis, QM predicts that the interference pattern will change when the solenoid is switched on and off and, in particular, the fringe shift will show a systematic dependence on the total magnetic flux contained in $\mathcal{S}_\infty$. This is shocking to intuitions trained on classical mechanics which teaches that the behavior of a charged particle depends only on the electromagnetic field it encounters along its trajectory. QM apparently undoes this lesson, at least under the idealizations (F1)-(F3) which imply that the electrons never encounter a region where $\boldsymbol{B}_\infty$ is non-zero.

The latter fact can be verified by using classical electromagnetism and the Coulomb gauge to compute the components of the vector potential $\boldsymbol{A}_\infty$ for an infinitely long cylindrical solenoid of radius R. In polar coordinates (ρ, z, θ), $\rho := (x^2 + y^2)^{1/2}$, where the z-axis is chosen as the axis of the cylinder, the components of $\boldsymbol{A}_\infty$ take the following form:

$$
\begin{aligned}
(\boldsymbol{A}_\infty)_z &= (\boldsymbol{A}_\infty)_\rho = 0 \qquad (1)\\
(\boldsymbol{A}_\infty)_\theta(\rho) &= \frac{\Phi_\infty}{2\pi\rho} \text{ for } R \leq \rho \\
&= \frac{\Phi_\infty \rho}{2\pi R^2} \text{ for } 0 \leq \rho \leq R
\end{aligned}
$$

where Φ_∞ is the magnetic flux through $\mathcal{S}_\infty$. As expected, $\boldsymbol{B}_\infty = \nabla \times \boldsymbol{A}_\infty = 0$ in the region $\mathbb{R}^3 \backslash \mathcal{S}_\infty$ exterior to the solenoid, which under idealizations (F1) and (F2) is the electron configuration space (i.e. the part of physical space $\mathbb{R}^3$ accessible to the electron). Alternatively, one could start with the realistic case of a solenoid $\mathcal{S}_L$ of finite length, $L < \infty$, solve for $\boldsymbol{A}_L$, and verify that $(\boldsymbol{A}_L)_\theta(\rho)$ converges pointwise to $(\boldsymbol{A}_\infty)_\theta(\rho)$ as $L \to \infty$ (de Oliveira and Pereira 2008).

In addition to the magnetic AB effect outlined above, there is also an electric AB effect. However, both the theoretical analysis and the experimental verification of the latter are still being discussed (see Weder 2011,

[9] "[I]nfinitely repulsive barriers do not really exist" (Magni and Valz-Gris 1995).

Eskin 2013, Wang 2015, and the references therein). Here I concentrate on the magnetic effect, so when I speak of *the* AB effect I mean the magnetic effect.

3 Some background for the analysis of the AB effect

3.1 Stokes' theorem

Stokes' theorem is standardly invoked in discussions of the AB effect to conclude that for a closed path γ that is the boundary of a surface S

$$\Phi(S) := \int_S \boldsymbol{B}\cdot\boldsymbol{dS} = \oint_\gamma \boldsymbol{A}\cdot\boldsymbol{dx}, \tag{2}$$

where $\boldsymbol{dS}$ stands for a surface element and $\Phi(S)$ is the magnetic flux through the surface S. The theorem asserts that given a continuously differentiable vector field $\boldsymbol{F}$ in a region $\mathcal{R}$ of space and a closed path γ in $\mathcal{R}$, if γ is the boundary of a two-sided surface S lying entirely in $\mathcal{R}$, then $\int_S \boldsymbol{\nabla}\times\boldsymbol{F}\cdot\boldsymbol{dS} = \oint_\gamma \boldsymbol{F}\cdot\boldsymbol{dx}$. It follows from this theorem that if $\boldsymbol{\nabla}\times\boldsymbol{F} = 0$ then $\boldsymbol{F} = \boldsymbol{\nabla}f$, where f is a scalar field on $\mathcal{R}$, a consequence that will be used repeatedly below. If γ lies in a simply connected region $\mathcal{R}$ then Stokes' applies since it is guaranteed that γ is the boundary of a two-sided surface S lying entirely in $\mathcal{R}$. Since physical space is assumed to be $\mathbb{R}^3$, and is therefore simply connected, Stokes' theorem can be combined with the idealization (F2) to conclude that for a closed path γ encircling the solenoid, $\oint_\gamma \boldsymbol{A}\cdot\boldsymbol{dx} = \Phi_\infty$, a result that can also be obtained directly from (1) by integration. However, Stokes' theorem cannot be applied within the non-simply connected electron configuration space $\mathcal{R} = \mathbb{R}^3 \backslash \mathcal{S}_\infty$.[10] In Section 6 I will consider what would constitute the AB effect for an embodied physicist who, like the electron, is confined to $\mathcal{R} = \mathbb{R}^3 \backslash \mathcal{S}_\infty$. But until then I will examine the AB effect from the God's eye perspective in which all of physical space can be surveyed.

[10] The line integral of $\boldsymbol{A}_\infty$ for a closed path in $\mathbb{R}^3 \backslash \mathcal{S}_\infty$ encircling the solenoid is non-zero whereas $\int_S \boldsymbol{\nabla}\times\boldsymbol{A}\cdot\boldsymbol{dS} = 0$ for any surface lying wholly in $\mathbb{R}^3 \backslash \mathcal{S}_\infty$.

3.2 Non-simple connectedness

A good deal of attention in the philosophical literature on the AB effect focuses on the non-simple connectedness of the electron configuration space. In one sense there is a good reason for this focus, at least if the AB effect is understood to require a strictly null intersection between the electron configuration space and the region of physical space where $\boldsymbol{B} \neq 0$. For if the electron configuration space were simply connected then Stokes' theorem would be applicable, and together with $\boldsymbol{B}_{con} = \nabla \times \boldsymbol{A}_{con} = 0$ the theorem implies that $\boldsymbol{A}_{con} = \nabla f_{con}$ for some scalar field f_{con} on the electron configuration space so that $\boldsymbol{A}_{con}$ can be set to zero by appropriate choice of gauge. The Hamiltonian operator for the electron in the presence of a magnetic field is taken to have the form

$$H = (\boldsymbol{p} - \frac{e}{c}\boldsymbol{A})^2, \quad \boldsymbol{p} = -i\boldsymbol{\nabla} \tag{3}$$

where, e is the charge of the electron and units have been chosen so that the mass of the electron is $1/2$ and $\hbar = 1$. Thus, the result in question means that if the electron configuration space were simply connected, the Hamiltonian can be gauge transformed into a form $H_0 = \boldsymbol{p}^2$ that describes a free electron.

But it is at this juncture that two of the deficiencies in the philosophical literature begin to reveal themselves. First, having arrived at the relation $\oint_\gamma \boldsymbol{A} \cdot \boldsymbol{dx} = \Phi_\infty$ for a closed path γ lying in the the electron configuration space $\mathbb{R}^3 \backslash \mathcal{S}_\infty$, the impression is sometimes given that there is a simple and direct way to conclude that the phase shift observed in the set up in Fig. 1 is proportional to Φ_∞. For example, typical presentations assert, as if it were a self-evident truth, that in the AB set up the electron wave function acquires a phase difference proportional to $\boldsymbol{A} \cdot \boldsymbol{dx}$ as it moves from $\boldsymbol{x}$ to $\boldsymbol{x} + \boldsymbol{dx}$ (see, for example, Healey 2007, p. 24). But there is no magic direct route to the wanted conclusion. The problem to be solved is a dynamical one: the phase shift has to be deduced by comparing the solution to the Schrödinger equation when the solenoid is turned on with the solution for when the solenoid is turned off. The dynamical analysis reveals that the supposed self-evident truth is an *approximate* truth, which can be made valid to a high degree of accuracy for high-velocity wave packets (see Section 5.1). The dynamical analysis also reveals a second deficiency in the philosophical literature; namely, there is no recognition of the fact that the idealization with features (F1) and (F3) that give rise to the non-simple connectedness of the electron configuration space make the analysis of the dynamics complicated since these features have the consequence that the Hamiltonian operator for electron is not (essentially) self-adjoint, which stymies dynamical analysis until extra assumptions are imported.

Before turning to the details of the dynamics a word needs to be said about early attempts to show that the AB effect is not a genuine physical effect.

3.3 Attempt to dismiss the AB effect

Under the idealizations (F1)-(F3) the Hamiltonian operator (3) for the electron contains the gauge dependent quantity $\boldsymbol{A}_\infty$, and it needs to be checked that the prediction of the AB effect does not depend on the choice gauge for the vector potential. It was once claimed that this check fails. Two vector potentials $\boldsymbol{A}$ and $\boldsymbol{A}'$ are gauge equivalent (i.e. correspond to the same $\boldsymbol{B}$ field) iff $\boldsymbol{A}' = \boldsymbol{A} + \boldsymbol{A}''$ where $\boldsymbol{\nabla}\times\boldsymbol{A}'' = 0$. If Stokes' theorem applies, it follows that $\boldsymbol{A}'' = \boldsymbol{\nabla} f$. Consider the transformation of the vector potential $\boldsymbol{A}_\infty \mapsto \boldsymbol{A}'_\infty = \boldsymbol{A}_\infty + \boldsymbol{\nabla} f$ with $f := -\dfrac{\Phi_\infty \theta}{c}$.[11] In this new gauge formula (1) is replaced by

$$
\begin{aligned}
(\boldsymbol{A}'_\infty)_\theta &= 0 \quad \text{for } R \leq \rho \\
&= \frac{\Phi_\infty \rho}{2\pi R^2} - \frac{\Phi_\infty}{2\pi\rho} \quad \text{for } 0 \leq \rho \leq R.
\end{aligned}
\tag{1*}
$$

In a paper entitled "Nonexistence of the Aharonov-Bohm Effect" Bocchieri and Loinger (1978) argued since the vector potential for the region accessible to the electron has been gauge transformed away and the Hamiltonian assumes the form for a free electron, the AB effect has a "purely mathematical origin" and, consequently, cannot be a genuine physical effect. Their argument is undermined by the fact that the new gauge is "non-Stokesian"; that is, since $\boldsymbol{A}'_\infty$ is not continuously differentiable and Stokes' theorem does not apply. For further comments on the inadmissibility of non-Stokesian gauges see Klein (1979) and Bohm and Hiley (1979).

Bocchieri and Loinger (1978) sought to buttress their case against the reality of the AB effect by reference to the hydrodynamical reformulation of

[11] This transformation will appear in a new guise in Section 5.1.

QM, wherein the state of the system is described by the probability density $\rho := \psi\psi^*$ and the current density $\mathbf{j} := -i(\psi^*\boldsymbol{\nabla}\psi - \psi\boldsymbol{\nabla}\psi^*) = \mathrm{Im}(\psi^*\boldsymbol{\nabla}\psi)$, and the Schrödinger equation for the wave function is replaced by a set of non-linear partial differential equations governing the evolution of ρ and $\mathbf{j}$. Since these latter equations employ only the electromagnetic fields and not the potentials, Bocchieri and Loinger concluded that "they leave no room for the effects of the kind of Aharonov's and Bohm's" (1978 p. 482; see also Bocchieri and Loinger 1981a). Two uses of the hydrodynamical reformulation of QM should be distinguished: to argue against the existence of the AB effect as a prediction of QM under the idealization (F1)-(F3) (or alternative idealizations discussed below) vs. to argue that the phase shifts detected in real world experiments do not count as confirmation of the AB effect because the hydrodynamical formulation shows that phase shifts in these experiments can be explained in terms of the local interaction of the electron with the magnetic field. As will be discussed below (see Section 7) other researchers besides Bocchieri and Loinger fell in with the latter usage. But the former usage is problematic since, under the idealization (F1)-(F3), where the configuration space of the electron is not simply connected, the formal equivalence between the Schrödinger equation and the hydrodynamical equations of motion is lost, making the comparison between standard QM and the hydrodynamical reformulation difficult (see Casati and Guarneri 1979). In any case, if standard QM does predict the AB effect under the idealization (F1)-(F3) and the hydrodynamical formulation does not, then (one might say) so much the worse for the latter since all bets should be on the former.

This makes it all the more important to understand in what sense standard QM predicts the AB effect. That is the goal of the next two sections.

4 The AB effect from the God's eye perspective: an initial stumbling block

4.1 The Hamiltonian operator and its self-adjoint extensions

From the God's eye perspective from which all of physical space can be surveyed, under the idealizations (F1)-(F3) the Hamiltonian operator for the electron has the form $H^{\boldsymbol{A}_\infty} := (\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}_\infty)^2$, where $\boldsymbol{A}_\infty$ is given by eq. (1).

Since it is unbounded, this differential operator expression does not specify a Hilbert space operator until its domain is specified. To count as an observable, the operator with specified domain must be self-adjoint[12], or at least essentially self-adjoint (i.e. there is a unique extension to a larger domain on which the operator is self-adjoint). If the Hamiltonian operator passes this check then exponentiating the unique self-adjoint extension yields a strongly continuous unitary group that supplies the dynamics for the electron, the Schrödinger equation being the infinitesimal version. Under the idealization (F1)-(F3) the configuration space for the electron is $\mathbb{R}^3 \backslash \mathcal{S}_\infty$ and the Hilbert space is $\mathcal{H} = L^2(\mathbb{R}^3 \backslash \mathcal{S}_\infty)$.[13] The natural (initial) domain for $H^{\boldsymbol{A}_\infty}$ is $D(H^{\boldsymbol{A}_\infty}) = C_0^\infty(\mathbb{R}^3 \backslash \mathcal{S}_\infty)$ which is dense in $L^2(\mathbb{R}^3 \backslash \mathcal{S}_\infty)$.[14] With this choice $H^{\boldsymbol{A}_\infty}$ *does not correspond to an observable* since it is *not* essentially self-adjoint on $D(H^{\boldsymbol{A}_\infty})$ and, thus, does *not* generate the dynamics.

Now since $H^{\boldsymbol{A}_\infty}$ is symmetric on $D(H^{\boldsymbol{A}_\infty})$ and since it commutes with complex conjugation it does have self-adjoint extensions. The multiplicity of these extensions depends on the choice of two ways of specializing idealization (F1): either (F1a) where $\mathcal{S}_\infty$ is taken to have a finite radius $R > 0$, or the more severe idealization (F1b) where the radius of $\mathcal{S}_\infty$ is shrunken to zero without affecting the value of the flux, leaving a thread of magnetic flux along the z-axis. In case (F1b) the deficiency indices of $H^{\boldsymbol{A}_\infty}$ are both 2 while in case (F1a) they are both ∞. The deficiency indices are the dimensions of the deficiency spaces, and the self-adjoint extensions are in one-one correspondence with the unitary maps between the deficiency spaces.[15] For the idealization (F1b) the self-adjoint extensions of $H^{\boldsymbol{A}_\infty}$are in one-one correspondence with 2x2 unitary matrices, which are parametrized by four real numbers and, thus, and there is a four-fold infinity of self-adjoint extensions (Adami and Teta 1995). For the idealization (F1a) there is an infinity-fold infinity of self-adjoint extensions (de Oliveira and Pereira 2010). The different self-adjoint extensions correspond to different boundary conditions on the wave function at $\rho = 0$ for idealization (F1b) and at $\rho = R > 0$ for

[12] A linear operator A on a Hilbert space is self-adjoint iff A is symmetric (a.k.a. Hermitian) and A's domain coincides with the domain of the adjoint of A.

[13] $L^2(X)$ denotes the Hilbert space of complex valued square integrable functions on X. No explicit reference to measure is needed since in all the applications considered Lesbegue measure is used.

[14] $C_0^\infty(X)$ stands for the smooth functions of compact support on X. This is the standard "test function" space used in rigorous mathematical treatment of Hilbert space operators.

[15] For the relevant mathematical background, see Reed and Simon (1975, Section X.1).

idealization (F1a).[16]

The point to emphasize is that the different self-adjoint extensions of $H^{\boldsymbol{A}_\infty}$ produce different physics; in particular, energy eigenvalues will be different in different extensions, and the unitary (Schrödinger) dynamics obtained by exponentiating the different self-adjoint extensions will be different. The different scattering cross sections for different extensions in the (F1a) idealization are studied in de Oliveira and Pereira (2010).

On the traditional way of thinking about idealizations, one of the supposed benefits of idealization is simplification and tractability of analysis. But in the present instance the simplification achieved by applying the idealizations (F1)-(F3) hides a seething complexity in the different ways the Hamiltonian operator can be made self-adjoint. Another, more idealization friendly, spin on this finding is to say that what it reveals is that the idealizations (F1)-(F3) do not produce a model of ordinary QM but rather a model schema that can be turned into a concrete model with a self-adjoint Hamiltonian in an infinity of physically inequivalent ways. Now the question becomes: Which of these many inequivalent concretizations of the model schema is to be used, and why?

4.2 Choosing a self-adjoint extension

The different self-adjoint extensions of the Hamiltonian operator $H^{\boldsymbol{A}_\infty}$ correspond to different boundary conditions on the wave function at the border of the solenoid, and the different boundary conditions can in turn be thought of as representing different ways the electron can interact with the solenoid border. For continuously differentiable wave functions the impenetrability assumption requires a vanishing at the solenoid border of the normal component $\mathbf{j}_N$ of the electron probability current density. By inspection, there are various sufficient conditions for meeting this requirement, the most obvious being $\psi = 0$ (Dirichlet boundary conditions), $\boldsymbol{\nabla}_N \psi = 0$ (Neumann boundary conditions), or $\boldsymbol{\nabla}_N \psi = r\psi$, $r \in \mathbb{R}$, (Robin boundary conditions). These are sufficient but not necessary conditions, and there are many other

[16] The reader who wants to get a feel for what is going on here may wish to start with the toy example in Reed and Simon (1975, Section X.1). The Hamiltonian operator $-d^2/dx^2$ for a free particle moving on the truncated real line (R, ∞), $R > 0$, is not essentially self-adjoint on the domain $C_0^\infty(R, \infty)$. There is a one-parameter infinity of self-adjoint extensions corresponding to different boundary conditions on the wave function as $x^+ \to R$.

ways to guarantee the vanishing of $\mathbf{j}_N$ at the solenoid boundary. Aharonov and Bohm (1959) used the idealization (F1b) (thread of magnetic flux) and assumed that the wave function vanishes as $\rho \to 0$. In this subsection I will work with (F1a) (infinitely long solenoid with radius $R > 0$). In this case the Aharonov-Bohm Hamiltonian is the self-adjoint extension $\overline{H}^{\boldsymbol{A}_\infty}_{AB}$ of $H^{\boldsymbol{A}_\infty}$ corresponding to Dirichlet boundary conditions where the electron wave function vanishes at the border $\rho = R > 0$ of the solenoid. What then is the justification for this choice of self-adjoint extension? A possible answer is provided by a very pretty mathematical result by Oliveira and Pereira (2008).[17]

Start with a sequence of more realistic Hamiltonians arising from finite length solenoids surrounded by potential barriers of finite height:

$$H_{L,n} = (\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}_L)^2 + V_{L,n} \tag{4}$$

where $\mathbf{A}_L$ is the vector potential for a solenoid of length $0 < L \leq \infty$ and $V_{L,n}(x) := n\chi_L(x)$, $n = 0, 1, 2, ...$, where χ_L is the characteristic function for the interior of the cylinder $\mathcal{S}_L$ (i.e. $\chi_L(x) = 1$ if $x \in int(\mathcal{S}_L)$ and 0 otherwise). The positive potential $V_{L,n}$ acts as a repulsive barrier to electrons; but with $n < \infty$ the barrier is not impenetrable since electrons can tunnel through. Thus, for $n < \infty$ the electron configuration space is the full $\mathbb{R}^3$, the $H_{L,n}$ act on the Hilbert space $L^2(\mathbb{R}^3)$ and each of these Hamiltonians *is* essentially self-adjoint on $C_0^\infty(\mathbb{R}^3)$, which is dense in $L^2(\mathbb{R}^3)$—so there is nothing untowards for any finite L and n. As the length L of the solenoid is increased without bound the idealization (F1a) is approached, and as the height n is the potential barrier is increased without bound the idealization (F2) is approached. De de Oliveira and Pereira (2008) prove that in the idealized limit in which $L \to \infty$ and $n \to \infty$, $H_{L,n}$ converges to the Aharonov-Bohm Hamiltonian $\overline{H}^{\boldsymbol{A}_\infty}_{AB}$ in the strong resolvent sense, independently of order in which the limits are taken.[18]

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[17]Shech (2015, 2017) argues that the standard account of the AB effect is flawed because it offers no satisfying justification for choosing the Dirichlet boundary conditions. I agree. But I argue below that this choice makes no difference for the predicted phase shift; where it does make a difference in predictions for scattering of electrons off the cylinder. And for reasons given below I do not think that, by themselves, the results of de Oliveira and Pereira (2008) justify the choice of Dirichlet boundary conditions. But I emphasize that the de Oliveira and Pereira (2008) results are important for the confirmation of the AB effect (see Section 8).

[18]What strong resolvent convergence means is roughly this. Consider a sequence $\{H_m\}$ of self-adjoint Hamiltonian operators on $L^2(\mathbb{R}^3)$ and the associated one parameter groups

A justification for the Aharonov-Bohm Hamiltonian requires joining to the de Oliveira-Pereira result some additional assumptions about how the idealization with the features (F1a), (F2) and (F3) is realized. For example, one could imagine that the idealization is achieved by means of the following supertask: in the first minute a solenoid of length 1 and a step-function potential of height 1 at the solenoid border are erected; in the next 1/2 minute the solenoid is extended to length 2 and the potential barrier is raised to height 2; etc. ad infinitum. At the end of two minutes the infinite sequence of building tasks is completed, resulting in the sought after idealization. Applying the Oliveira-Pereira mathematical result shows that Dirichlet boundary conditions and the Aharonov-Bohm Hamiltonian are a by-product. But this kind of argumentation threatens to make the choice of self-adjoint extension dependent on this history of the fictional system; for it is not implausible to conjecture that there are other supertasks using a different sequence of finite-height repulsive potentials of different shapes that eventuate in the sought after idealization but produce in the limit different boundary conditions and, thus, different self-adjoint extensions of the Hamiltonian operator. And in any case, there is the alternative boring history in which no supertasks are performed and in which an infinitely high potential barrier has always and forever surrounded an infinite length solenoid. This history provides no guidance for what boundary conditions or self-adjoint extension should be used. Needless to say, if the AB effect depends on which fictional history gives rise to the idealization (F1a), (F2), (F3) then implications of the effect for foundations of physics have to be tempered accordingly.

Apart from whatever role results like that of de Oliveira and Pereira do or do not play in justifying a particular self-adjoint extension, they are important to the confirmation of the AB effect; for they can be used as part of an argument to show that actual world experiments, which perforce fall short of the fictional world conditions that characterize an AB system, can lend support to predictions of the quantum theory about this fictional system. This matter will be taken up in Section 7. But first it is high time to take up the details of how the AB effect is supposed to arise in the idealization (F1a), (F2), (F3).

$\{U_m(t)\}$ obtained by exponentiating the H_ms. Suppose that there is a unitary group $U(t)$ such that $\psi(t) := U(t)\psi_0 = \lim_{m\to\infty} U_m(t)\psi_0$ for all $\psi_0 \in L^2(\mathbb{R}^3)$. Then the strong resolvent limit H_∞ of $\{H_m\}$ can be defined as the generator of $U(t)$.

5 Deriving the phase shift

The mathematical formalism used below should not cause one to lose focus on the question that the formalism is supposed to help us answer: What does QM say would happen under the idealization (F1)-(F3) when the solenoid is switched on and off? Even with the help of the formalism of QM one runs into some of the problems, all too familiar to philosophers, with trying to evaluate counterfactual scenarios. I begin with reviewing the standard derivation of the quantitative expression for the phase shift. Afterwards I will evaluate how well the derivation serves to answer the question at issue.

5.1 Explaining the AB phase shift

Standard derivations of the AB phase shift for the setup of Fig. 1 suppose that the wave function of the electron is in the form of a wave packet that, after leaving the source, is split into two parts that reach the screen by different routes labeled γ_1 and γ_2. The goal is to compute the additional phase difference in the components of the electron wave function that results when the solenoid is switched on.

Towards this end consider the formal expression

$$U(x) := \exp(i\frac{e}{c}\int_{x_0}^{x} \boldsymbol{A}(\boldsymbol{x}') \cdot \boldsymbol{dx}') \tag{5}$$

where $x, x_0 \in \mathbb{R}^3\backslash\mathcal{S}_\infty$ and x_0 is a fixed point. If the integral in the exponent were independent of the contour of integration then (5) would define a unitary map that could be used to transform away the vector potential term in the electron Hamiltonian, reducing it to the free electron Hamiltonian. To see this, use polar coordinates in which the God's eye perspective electron Hamiltonian takes the form

$$H^{\Phi_\infty} = p_\rho^2 + \frac{1}{\rho^2}(p_\theta - \frac{e\Phi_\infty}{c})^2 + p_z^2, \;\; p_\theta := -i\partial/\partial\theta \tag{6}$$

and in which (5) assumes the form

$$U(\rho, \theta, z) = \exp(i\frac{e\Phi_\infty\theta}{c}). \tag{7}$$

Note that—formally at least—$Up_\theta U^{-1} = p_\theta + \dfrac{e\Phi_\infty}{c}$ and $UH^{\Phi_\infty>0}U^{-1} =$

$H^{\Phi_\infty=0}$. But with $x, x_0 \in \mathbb{R}^3\backslash\mathcal{S}_\infty$ the integral in the exponent of (5) is not independent of the contour of integration and the transformation $U(\rho,\theta,z)$ is singular unless $e\Phi_\infty/c$ is an integral multiple of 2π—$U(\rho,\theta,z)$ is discontinuous for some θ^* as $\theta^+ \to \theta^*$ and $\theta^- \to \theta^*$.[19]

By means of an Ansatz that uses the transformation (5) localized to simply connected subregions of $\mathbb{R}^3\backslash\mathcal{S}_\infty$ where it is non-singular, Aharonov and Bohm (1959) supplied a semi-classical approximation to obtain the phase shift produced by switching on the solenoid. For sake of concreteness, choose the self-adjoint extensions $\overline{H}^{\Phi_\infty=0}$ and $\overline{H}^{\Phi_\infty>0}$ respectively for $H^{\Phi_\infty=0}$ and $H^{\Phi_\infty>0}$ picked out by Dirichlet boundary conditions. For a simply connected subregion $\mathcal{R}_1 \subset \mathbb{R}^3\backslash\mathcal{S}_\infty$ that surrounds path γ_1, $\Lambda_1(x) := \int_{x_0}^{x} \frac{ie}{c}\boldsymbol{A}(\boldsymbol{x}')\cdot \boldsymbol{dx}'$, $x_0, x \in \mathcal{R}_1$, is independent of the contour from the fixed point x_0 to x; and similarly for a simply connected region $\mathcal{R}_2 \subset \mathbb{R}^3\backslash\mathcal{S}_\infty$ that surrounds path γ_2. Aharonov and Bohm (1959) made an Ansatz asserting that an exact solution $\psi_{1,\Phi_\infty>0}(x,t) = \exp(-it\overline{H}^{\Phi_\infty>0})\phi_1(x,0)$ of the unitary dynamics for $\overline{H}^{\Phi_\infty>0}$ (solenoid switched on) with initial conditions $\phi_1(x,0)$ at $t=0$ that produce a wave packet concentrated on γ_1 is well approximated by the multiplication of the solution $\psi_{1,\Phi_\infty=0}(x,t)$ of the $\overline{H}^{\Phi_\infty=0}$ (solenoid switched off) dynamics by the magnetic factor $\exp(-\Lambda_1(x))$, i.e. the exact switched on solution is well approximated by

$$\begin{aligned}\psi_{AB,1}(x,t) &:= \exp(-\Lambda_1(x))\psi_{1,\Phi_\infty=0}(x,t) \\ &:= \exp(-\Lambda_1(x))\exp(-it\overline{H}_\infty^{\Phi_\infty=0})\phi_1(x,0);\end{aligned} \tag{8}$$

and similarly for the wave packet $\psi_{2,\Phi_\infty>0}(x,t)$ concentrated on γ_2.[20]

The upshot of the AB Ansatz is that the total wave packet $\psi_{AB}(x,t) = \psi_{1,\Phi_\infty>0}(x,t) + \psi_{2,\Phi_\infty>0}(x,t)$ for the $\overline{H}_\infty^{\Phi_\infty>0}$ (solenoid switched on) evolution should be well approximated by

$$\begin{aligned}\psi_{AB}(x,t) &= \exp(-\Lambda_1(x))\psi_{1,\Phi_\infty=0}(x,t) + \exp(-\Lambda_2(x))\psi_{2,\Phi_\infty=0}(x,t) \\ &= \exp(-\Lambda_1(x))\{\psi_{1,\Phi_\infty=0}(x,t) + \exp(\Lambda_1(x)-\Lambda_2(x))\psi_{2,\Phi_\infty=0}(x,t)\}.\end{aligned} \tag{9}$$

[19] The discontinuity should not be conflated with so-called multi-valuedness.

[20] The wave packet of the electron can, to good approximation, remain within the connected region $\mathcal{R}_1$ (or $\mathcal{R}_2$), but the tails of the wave packet will be spread throughout the electron's configuration space.

Choosing x_0 to be at the source of the electrons and choosing the contours for the Λ_1 and Λ_2 integrals to be along γ_1 and γ_2 respectively gives

$$\begin{aligned}\psi_{AB}(x,t) = \exp(-\Lambda_1(x))\{\psi_{1,\Phi_\infty=0}(x,t) + \exp(\int_{\gamma_1} \frac{ie}{c}\boldsymbol{A}(\boldsymbol{x}')\cdot \boldsymbol{dx}' \\ - \int_{\gamma_2} \frac{ie}{c}\boldsymbol{A}(\boldsymbol{x}')\cdot \boldsymbol{dx}')\psi_{2,\Phi_\infty=0}(x,t)\} \\ = \exp(-\Lambda_1(x))\{\psi_{1,\Phi_\infty=0}(x,t) + \exp(\oint_{\gamma} \frac{ie}{c}\boldsymbol{A}(\boldsymbol{x}')\cdot \boldsymbol{dx}')\psi_{1,\Phi_\infty=0}(x,t)\} \\ = \exp(-\Lambda_1(x))\{\psi_{1,\Phi_\infty=0}(x,t) + \exp(\frac{ie\Phi_\infty}{c})\psi_{2,\Phi_\infty=0}(x,t)\}\end{aligned} \tag{10}$$

where $\gamma = \gamma_1 - \gamma_2$, resulting in a prediction of a relative phase change of $\exp(\frac{ie\Phi_\infty}{c})$.[21] Unless $\frac{c\Phi_\infty}{e}$ is an integer multiple of 2π there should be a fringe shift. This prediction is gauge invariant since, although both of the factors $\exp(\pm\Lambda_1(x))$ are gauge dependent, $\exp(\Lambda_1(x))\exp(-\Lambda_2(x)) = \exp(\Lambda_1(x) - \Lambda_2(x))$ is gauge invariant.

The soundness of the derivation of the phase shift now boils down to the question of how good an approximation to the exact solution the AB Ansatz provides. It might seem surprising that only recently have rigorous investigations of this question appeared in the physics literature. For a different idealized system exhibiting the AB effect to be discussed below in Section 7, Ballesteros and Weder (2009, 2011) have provided—a half century after the seminal Aharonov-Bohm paper(1959)!—rigorous error bounds for the AB Ansatz, from which it follows that high velocity Gaussian wave packets do make the AB Ansatz a good approximation.

5.2 So does the choice of self-adjoint extension matter?

The above derivation of the phase shift was couched in terms of particular self-adjoint extensions of $H^{\Phi_\infty=0}$ and $H^{\Phi_\infty>0}$ corresponding to Dirichlet boundary conditions at the solenoid border. But nothing in the reasoning is limited to these particular extensions; the derivation would have worked just

[21] Manipulations of this sort can be found in many texts; see for example Nakamura (1990, 356-359). The cynic might say that this derivation looks like a case of seeing the result one wants and working backwards to an Ansatz that will generate it. There is nothing wrong with such a procedure as long as the Ansatz can be justified.

as well as applied to different self-adjoint extensions of $H^{\Phi_\infty=0}$ and $H^{\Phi_\infty>0}$ corresponding to, say, Neumann boundary conditions. All that is required is the assumption that the boundary conditions do not change when the solenoid is switched on and off. But unless additional physical principles or additional stories about the history of the realization of the fictional system are brought to bear, the assumption remains no more than an assumption. Philosophers who struggle with evaluating questions about counterfactual scenarios will not be surprised by how tricky it is to answer such questions even within the constraints provided by the quantum theory.

In any case the AB effect should not be construed so narrowly as to be exhausted by the flux dependent interference patterns observed on the screen in setups like that illustrated in Fig. 1. More broadly construed, the AB effect is any experimentally verifiable, systematic dependence of the behavior of electrons on the magnetic flux in situations where the electron never encounters regions where the magnetic field is non-zero.[22] Since different concretizations of the model schema of (F1)-(F3) (i.e. different self-adjoint extensions of $H^{\Phi_\infty=0}$ and $H^{\Phi_\infty>0}$) *do* involve different dependencies in the behavior of electrons on the magnetic flux, the broadly construed AB effect is not a universal effect but is model specific.

A concrete example of the point is supplied by scattering of electrons off the idealized solenoid. One advantage of this scenario is that exact solutions can be obtained without the need to rely on approximations, Ansätze, or appeals to physical plausibility. Aharonov and Bohm (1959) derived the scattering cross section for the case of a thread of magnetic flux (idealization (F1b)) using Dirichlet boundary conditions, but their analysis was criticized by Feinberg (1963) and Henneberger (1981). A more solid analysis of this case was given by Ruijenaars (1983) also assuming Dirichlet boundary conditions. But more important for our purposes is the analysis of de Oliveira and Pereira (2010) for the case of solenoid of finite radius. Their analysis shows how the scattering cross section for this case depends on the boundary conditions at the border of the solenoid and, thus, on the self-adjoint extension of $H^{\Phi_\infty>0}$. In sum, the AB effect, broadly construed, *does* depend on the concretization of the idealization schema (F1)-(F3). "The AB effect" is, therefore, a bit of

[22] An example of this broad understanding is to be found in Eskin (2013) where the AB effect is taken to mean that there are solutions to the Schrödinger dynamics where "a physical quantity such a probability density ... or probability current ... depends on the gauge equivalence class of the magnetic potential" in situations where the magnetic field vanishes in the electron's configuration space.

a misnomer since it points to a family of effects.

It would be sad to have to conclude that the basic question—What does QM say would happen under the idealization (F1)-(F3) when the solenoid is switched on and off?—is not well posed until the history of the realization of (F1)-(F3) is specified. More happily it can be said that while the *details* of the answer to the what-would-happen question may depend on how the details of the what-if scenario are filled in, the *existence* of observable effects in the behavior of the electron reflecting the strength of the magnetic flux inside the solenoid do not so depend. Switching the solenoid on and off under the idealization (F1)-(F3) corresponds to *some* difference in the self-adjoint extension of the electron Hamiltonian, which in turn will produce some experimentally detectable effects—in phase shifts, scattering, or others. The correspondence may be one-many until further details are supplied, but existence of some difference is ensured. Those who find the assurance of such a weak sense of the AB effect disappointing can console themselves with the reflection that something is better than nothing.

5.3 An afterword on the role of non-simple connectedness

To repeat, the non-simple connectedness of the electron configuration space is essential to a strictly null intersection between the electron configuration space and the region of space where the magnetic field is non-zero (recall Section 3.2), a defining condition of the AB effect. In addition, non-simple connectedness is crucial to the existence of unitarily inequivalent representations to be discussed in the next Section.

However, it is important to note that neither non-simple connectedness nor the idealization which secures this feature is a "difference maker" in the production of the phase shifts or the other phenomena that are characteristic of the AB effect; for otherwise there could be no convincing confirmation of the AB effect from actual world experiments. To take a specific example, consider again the setup in Fig. 1. Maintain idealizations (F1) and (F2) (infinitely long solenoid that does not leak flux) but drop (F3) (complete impenetrability of the solenoid) in favor of a finite step-function potential barrier. The electron Hamiltonian will be the self-adjoint operator $H_{\infty,n}$ of eq. (4) for some finite value of n and $L = \infty$. No matter how well concentrated the two components of the electron wave packet are on the paths γ_1

and γ_2, the tails of the wave packet will spread to the solenoid[23] and, thus, there will be a non-zero probability that the electron will tunnel through the potential barrier and enter the solenoid, making the electron configuration space the full, simply connected $\mathbb{R}^3$. But the derivation of the phase shift goes through exactly as before—the derivation doesn't care whether or not the electron configuration space is simply connected. Of course, when the electron configuration space is simply connected the phase shift is not counted as an AB effect; but this is a semantic point, not a point about the factors responsible for the phase shift. Compare: 'If the electron configuration space were simply connected then the electron would not exhibit an AB effect (requiring by definition that the electron's configuration space is strictly disjoint from the the region where $\mathbf{B} \neq 0$) vs. 'If the electron configuration space were simply connected then the interference pattern would not exhibit the phase shift characteristic of the AB effect.' The first assertion is true by definition of the AB effect while the second is simply false. The point that while the non-simple connectedness plays no causal role in producing the observed effects, QM predicts that these effects persist in the presence of non-simple connectedness.

Some commentators have proposed to understand the AB effect in terms of a fibre bundle formalism (see Batterman 2003) and the non-trivial holonomies associated with a non-simply connected base space. Here there is a disconnect between two senses of scientific explanation: explanation as unification where the fibre bundle approach gains traction by uniting the AB effect with other effects vs. explanation as revealing key causal features where the fibre bundle approach is silent. Much more needs to be said about the challenges that the AB effect poses for accounts of scientific explanation, but that will have to await another occasion.

6 The AB effect from the perspective of an embodied physicist (or an AB effect without any magnetic field)

Thus far the AB effect has been discussed from the God's eye perspective from which all of physical space can be surveyed. But it is also worthwhile

[23]Recall that according to ordinary non-relativistic QM—which is assumed in most of the discussions of the AB effect—the wave function spreads infinitely fast.

to consider the perspective of an embodied observer who, under the idealization of an infinitely high potential barrier surrounding an infinitely long cylindrical region of space, is confined to making measurements in the region exterior to the cylinder. Such an observer faces a familiar case of underdetermination: there are many possible causes hidden within the interior region of the solenoid (including of course a magnetic flux) that could explain the behavior of the electrons she observes in the exterior.[24] Here I want to report an interesting twist showing that there is also an explanation that refers purely to the exterior region and does not invoke causes hidden behind the veil of the infinite potential barrier.

In preparation for this discussion, I will first report some results for the form of idealization (F1b) in which the radius of the cylinder tends to zero leaving the flux fixed, with the upshot being a tread of magnetic flux along the z-axis. The non-simple connectedness of the electron configuration space and space accessible to our embodied observer will be crucial to these results. Let $\mathbf{M}$ be a differentiable vector field defined on $\mathbb{R}^3\backslash\{z=0\}$ such that $\mathbf{M}_z = 0$ and $\nabla\mathbf{x}\mathbf{M} = 0$. For example, $\mathbf{M}$ might be the field

$$\mathbf{M} = (1/\sqrt{x^2+y^2})\mathbf{e}, \ \ \sqrt{x^2+y^2} > 0 \qquad (11)$$
$$\mathbf{e} \text{ the unit vector } (-y/\sqrt{x^2+y^2}, x/\sqrt{x^2+y^2}, 0).$$

Define $\mathbf{M}$-dependent momentum operators $\mathbf{P}$ by

$$\mathbf{P} = \boldsymbol{p} - \alpha\mathbf{M}, \ \alpha = const, \quad \boldsymbol{p} = -i\boldsymbol{\nabla}. \qquad (12)$$

It is easy to check that these momentum operators and the usual position operators (acting by multiplication) together satisfy the CCR, i.e. $[\boldsymbol{P}_x, \mathrm{x}] = i$, $[\boldsymbol{P}_x, \boldsymbol{P}_x] = [\boldsymbol{P}_x, \boldsymbol{P}_y] = 0$, etc. on a common dense domain of $L^2(\mathbb{R}^3\backslash\{z=0\})$. Since only a finite number of degrees of freedom are involved one might think that an appeal to the von Neumann uniqueness theorem for representations of the CCR would yield the consequence that these representations are all unitarily equivalent to the Schrödinger representation. However, the von Neumann uniqueness theorem requires the Weyl form the CCR (see Reed and

[24] It would be interesting to compare the underdetermination in this cases with the underdetermination is cosmological models where the event horizon of an observer is analogized to the infinite potential barrier in the AB setup.

Simon 1980, 274-275), and the exponentiated $\mathbf{M}$-dependent momentum operators can fail to conform to the Weyl CCR. In more detail, $\boldsymbol{P}_x$ and $\boldsymbol{P}_y$ are essentially self-adjoint on $C_0^\infty(\mathbb{R}^3\backslash\{z=0\})$. Denoting their unique self-adjoint extensions by a overbar, the Weyl CCR require that their exponentiations to unitary operators satisfy $\exp(is\overline{\boldsymbol{P}}_x)\exp(it\overline{\boldsymbol{P}}_y) = \exp(it\overline{\boldsymbol{P}}_y)\exp(is\overline{\boldsymbol{P}}_x)$ for all $s, t \in \mathbb{R}$. For the $\mathbf{M}$-field of eq. (11) this requirement fails unless $\frac{1}{\alpha}\oint_\gamma \boldsymbol{M}\cdot d\boldsymbol{x}$, with γ a closed path around the z-axis, is an integer multiple of 2π, and when it fails the representation in question is not unitarily equivalent to the Schrödinger representation (see Reeh 1988 and Arai 1992).[25]

Similar results carry over to the idealization (F1a). Let us then consider the point of view of an embodied observer who, like the electron, is confined to the exterior region $\mathbb{R}^3\backslash\mathcal{S}_\infty$ of the infinitely long cylinder of finite radius. Our observer detects no magnetic field and, therefore, naturally posits as the electron Hamiltonian the operator for a free-particle in the Schrödinger representation, viz. $H_0 = \boldsymbol{p}^2$, $\boldsymbol{p} = -i\boldsymbol{\nabla}$. Unfortunately, the observed behavior of the electron does not accord with this posit; more specifically, there is no self-adjoint extension of H_0 that yields the fringe shifts observed in interference experiments.[26] Since she is familiar with the literature of mathematical physics, our observer suspects that something subtle is behind her conundrum; in particular, she suspects that she is working with the wrong representation of the CCR. After a bit of calculation she discovers that accord with experiments is achieved if she uses the free particle Hamiltonian based on the momentum operator of eq. (12), viz. $\widetilde{H}_0 = \mathbf{P}^2 = (\boldsymbol{p} - \alpha\mathbf{M})^2$, with the value of the constant α adjusted appropriately.

Our observer may also notice that the vector field in eq. (11) is the Cartesian coordinate version of the electromagnetic vector potential given by eq. (1) for the region $\mathbb{R}^3\backslash\mathcal{S}_\infty$. Thus, her new Hamiltonian is the same form as the Hamiltonian that would be appropriate if, behind the impenetrable barrier separating her from the interior of the cylinder, there a magnetic flux Φ_∞ and α is equal to $\frac{e\Phi_\infty}{c}$. Her sympathies towards scientific realism tempt her to conjecture that there really is a magnetic field inside the cylinder and

[25] This should come as no surprise. For if $(\overline{\boldsymbol{P}}_x, \overline{\boldsymbol{P}}_y, \overline{\boldsymbol{P}}_z)$ were unitarily equivalent to $(\overline{p}_x, \overline{p}_y, \overline{p}_z)$ then the Hamiltonian $\overline{\mathbf{p}}^2$ would be unitarily equivalent to the Hamiltonian $\overline{\mathbf{P}}^2$ and, thus, since the former does not produce an AB effect neither should the latter, which is false.

[26] One of the subtleties here is that although $\boldsymbol{p}$ and $\mathbf{P} = \boldsymbol{p} - \alpha\mathbf{M}$ are essentially self-adjoint, their corresponding Hamiltonians are not.

that this is the explanation of her experimental results. The realist case is strengthened if different representations are required at different times since, the realist will urge, the most natural explanation is in terms of different values of Φ_∞ at the different times. But her instrumentalist conscience cautions that such a conjecture borders on metaphysical speculation since no experiment she can perform in the region to which she is confined can verify or refute the conjecture. Even if there were no magnetic field behind the impenetrable barrier the unitarily inequivalent representations of the CCR on $\mathbb{R}^3\backslash\mathcal{S}_\infty$ would still exist, giving AB-type effects without any magnetic field. Positing a hidden cause for why one rather than another of the inequivalent representations is in fact realized is tempting, but giving in to the temptation requires the leap of faith embodied in scientific realism.

Unitarily inequivalent representations also arise in another approach. Goldin et al. (1981) start from an insight of the proponents of the hydrodynamical formulation of QM; namely, it is the electron probability density and probability current, and not the wave function, that are genuine observables. One is then led to ask about representations of the commutation algebra of these observables, just as one asks about the representations of the CCR. For the idealization of an infinitely long cylinder $\mathcal{S}_\infty$ of finite radius Goldin et al. find that in the exterior region $\mathbb{R}^3\backslash\mathcal{S}_\infty$ the current algebra for the electron admits a one-parameter family of unitarily inequivalent representations. Again scientific realism would urge positing a cause, not directly observable to our observer, for why some particular representation of the commutation algebra fits with observable facts in the region $\mathbb{R}^3\backslash\mathcal{S}_\infty$; but again instrumentalism urges skepticism about such posits.

Apart from the issue of realism vs. instrumentalism issue, there is no mystery about the choice among the unitarily inequivalent representations of the CCR or the current algebra—it is dictated by the fit with the observed phase shift. It would be interesting to compare this case to the cases in QFT and the thermodynamic limit in models of phase transitions where there is also a choice among unitarily inequivalent representations; but this is a project for another occasion.

7 Experimental tests

Reports of experimental tests of the AB effect began appearing in 1960 (see Chambers 1960), the year following the publication of Aharonov and Bohm

(1959), but it was well into the 1980s before skeptics admitted that solid experimental confirmation had been established (or at least stopped publishing skeptical articles). The delay was due in part to an obvious conundrum. Fictional experimentalists inhabiting the fictional world of the idealizations (F1)-(F3) can build an apparatus fulfilling these idealizations and carry out the relevant tests of the predicted fringe shifts and scattering cross sections when electrons fired around or at the flux carrying device. But real world experimentalists do not have such luxuries; in particular, they have to operate with finite length, penetrable solenoids that leak magnetic flux. What conditions must an actual world apparatus satisfy in order that it can produce confirmation of the AB predictions for the fictional apparatus?

As will be detailed in the following section, much of the early discussion of the Aharonov-Bohm effect was in reaction to their suggestion that, in contrast to classical electrodynamics where the electromagnetic potentials play a merely auxiliary role, in quantum electrodynamics the potentials take on a physical significance. Thus, the skeptics demanded that for actual experiments to count as confirmation of the AB effect there should be no plausible way to attribute the actually observed effects to the interaction of the electron with the electromagnetic field in the region accessible to the electron. Strocchi and Wightman (1974) argued that there is such a way: No actual solenoid is protected by an infinitely high potential barrier, and even a tightly localized wave packet will have tails that spread instantaneously over the entire configuration space. Therefore,

> The solution of the Schrödinger equation always has a tail which runs into the region of nonvanishing field and that field, by purely local manifestly gauge-invariant action, produces the effect. It will not do to argue from finite propagation speed that the effect will not be felt elsewhere soon enough; in Schrödinger theory effects can be propagated instantaneously. (p. 2202)

By a "local manifestly gauge-invariant action" they are referring to the hydrodynamical reformulation of QM (recall Section 3.3).[27]

Another apparent road block to experimental confirmation, not relying on the hydrodynamical reformulation of QM, flows from a theorem due to Roy (1980). The theorem entails that for a finite length solenoid—which is

[27] But precisely because of the infinite propagation speed, it is not evident how the hydrodynamical formulation provides for local action. More on the locality issue below.

all actual world experimentalists can build—even if surrounded by an infinite impenetrable cylindrical barrier which confines the electron to the non-simply connected region exterior to the barrier, the electromagnetic potentials in the external region can be gauge transformed to an expression involving the fields in this region alone. (But for future reference, note Roy's expression for the electromagnetic potentials involves the fields nonlocally.)

It was realized almost immediately that Roy's apparent road block could be circumvented, albeit by means of another fictional system using a toroidal magnet instead of a cylindrical solenoid. The modified idealization has the features

(F4) A magnetic field completely contained in a toroidal region.

(F5) The torus is impenetrable to external electrons.

In a note added in proof, Roy (1980) reported that his theorem does not apply to this new idealization, and he attributed to A. S. Goldhaber and P. K. Kabir "stimulating observations on the possibility of an Aharonov-Bohm-type effect when electrons are confined outside a toroidal magnetic field" (p. 113). A proposal to test the AB effect by measuring the interference between electrons passing through the hole of the torus and those passing around it was published by Kuper (1980). Thirteen years prior Tassie (1963) had described the predicted scattering cross section for electrons moving in a plane bisecting the idealized torus of (F4)-(F5); no experimental test of the prediction was described, but the suggestion for a possible test was implicit.[28]

Actual experiments were soon to follow. Tonomura et al. (1982) detected a fringe shift for square-torodial magnet made of permalloy. However, Bocchieri and Loinger (1982) rejected the notion that the experimental result counted as confirmation of the AB effect since, they claimed, "the fringe shift is a mere effect of the Lorentz force on the portion of the electron wave going into the permalloy" (p. 371). A year earlier they had imagined an experiment on a toroidal solenoid, and essentially they repeated the Strocchi and Wightman (1974) line relying on the hydrodynamical reformulation of QM:

[28] When the donut hole of the torus is plugged by an impenetrable barrier, making the electron configuration space simply connected, the predicted scattering cross section does not depend on the amount of flux in the torus.

> [T]he time evolution of ρ [probability density] and $\mathbf{j}$ [current density]—and more generally, the expectation value of any physical observable—depends on the [electromagnetic] *field strengths* and not the potentials [according to the hydrodynamical formulation]. Therefore the internal magnetic field of the impenetrable torus does not produce any physical effect. If anything were observed, this would be due to an imperfect impenetrability or to the existence of stray fields. (Bocchieri and Loinger 1981b p. 449)[29]

Tonomura and coworkers (see Tonomura et al. 1986 and Osakabe et al. 1986) strove to achieve an experimental realization that closely approximated the ideal (F4)-(F5). The magnetic field of a toroidal magnet was confined by coating the magnet with a superconductor layer, taking advantage of the Meissner effect that quenches the magnetic field. The flux leakage, as measured by interference electron microscopy, was estimated at less than $h/20e$. A coating of copper on the magnet helped to prevent penetration of the electron. It was estimated that only a 10^{-6} portion of the incoming electron wave reached the magnetic field coherently. Did the detection of a phase shift under these nearly ideal conditions provide, as the authors claimed, "crucial evidence for the existence of the AB effect" (Tonomura et al. 1986 p. 794)?

In fact, after the publication of Tonomura et al. (1986) the naysaying on experimental confirmation of the AB effect died out. From the published literature I have been unable to determine why this is so. After all, the naysayers of the Strocchi-Wightman-Bocchieri-Loinger camp could have continued to argue that the near ideal conditions achieved in the Tonomura et al. experiment are not good enough and, indeed, never can be good enough; for in this, as in any actual experiment, there are stray fields and imperfect impenetrability, and this is enough to give purchase to the skeptical arguments mentioned.

Perhaps of significance is the fact that Tonomura et al. attempted to make such skepticism sound like unproductive anti-inductivism:

[29] The reader will recognize the running together of two uses of the hydrodynamical formulation of QM: to argue against he existence of the AB effect for idealized circumstances vs. to argue that in actual circumstances falling short of the ideal, detection of a phase shift does not provide confirmation of the AB effect.

> The most controversial point in the dispute over experimental evidence for the AB effect has been whether or not the phase shift would be observed when both electron intensity and magnetic field were extremely small in the region of overlap. Since experimental realization of absolutely zero [magnetic] field [in the electron configuration space] is impossible, the continuity of physical phenomena in the transition from negligibly small field to zero field should be accepted instead of perpetual demands for the ideal; if a discontinuity there is asserted, only a futile agnosticism results. (Tonomura et al. 1986, p. 794)

This appeal to the "continuity of physical phenomena in the transition from negligibly small field to zero field" would have been more convincing if it had been accompanied by the type of result of de Oliveira and Pereira (2008) showing that continuity considerations can be applied because the idealized AB system can be obtained as the limit of a sequence of actually realizable systems (recall Section 4.1). Of course, the skeptic could claim that there remains the possibility that the phase shift would eventually disappear if experiments are carried out far enough into the sequence with ever smaller flux leakage and ever diminishing penetration of the electron wave packet into the magnet. But this does seem like futile agnosticism that would stifle most inductive inference. Did Tonomura et al. implicitly assume that such results would be forthcoming? Did the critics relent because they perceived that continuity considerations were pushing their skepticism towards vulgar inductive skepticism? Perhaps unpublished sources and interviews will help to answer these questions, but this is a project for another occasion.

In closing I note that the critics could have pointed out that when the AB effect is implemented by idealizations (F1)-(F3) there is a discontinuity in "the transition from negligibly small field to zero field." In the limit in which the configuration space of the electron is strictly confined to the exterior of the solenoid by raising the potential barrier to an infinite height and the magnetic flux is strictly confined to the interior by making the solenoid infinitely long, there emerge unitarily inequivalent representations of the CCR on the exterior region. The behavior of the electron can be attributed to the choice of a representation of the CCR unitarily inequivalent to the Schrödinger representation assumed in usual treatment of the AB effect. No reference to a magnetic field confined to the solenoid is required, although the scientific realist may wish to posit such a field as the best explanation of the observed effects. It is plausible that a similar situation for the idealizations (F3)-(F4) for a toroidal magnet that are supposed to represent the limit of the experiments of Tonomura at al. (1986).

8 Some foundational issues

Recall that the title of Aharonov and Bohm (1959) is "Significance of Electromagnetic Potentials in Quantum Theory." Much of the initial critical reaction to the paper centered on the paper's suggestion about the status of these potentials in QM or, more precisely, in the bastardized theory in which a quantized electron is subjected an external classical electromagnetic field. In classical electromagnetism the electromagnetic potentials are gauge dependent quantities, i.e. they *overdescribe* the physics in the sense that their values correspond many-one to the intrinsic physical state, a gauge transformation being a transformation of the potentials that relate values of the potentials corresponding to the same physical state; and further, the potentials can be altogether eliminated from the laws of classical electromagnetism in favor of fields, whose values correspond one-one to the physical state. What the AB effect brought out in dramatic fashion is that the situation changes in the bastardized quantum-classical theory where it seems that the electromagnetic fields *underdescribe* the physics. But underdescribe in what way? What is the nature of the something more that is needed? Much of the initial critical reaction to the AB effect was not directed so much at the effect itself as at Aharonov and Bohm's attempt to answer these questions.

Aharonov and Bohm opined that in order to account for the AB effect it would

> [S]eem natural ... to propose that, in quantum mechanics, the fundamental physical entities are the [electromagnetic] potentials, while the fields are derived from the them by differentiation. (Aharanov and Bohm 1959, p. 490).

They commented that the obvious objection to this suggestion is that the bastardized theory, like classical electromagnetic theory, is gauge invariant, which seems to imply that the electromagnetic potentials "cannot have any meaning, except insofar as they are used mathematically, to calculate the fields" (ibid.). But then they retort that "We have seen from the examples described in this paper that [this implication] cannot be maintained" (ibid.).

They offer two suggestions for escaping the conundrum to which the AB effect seems to lead:

> First, we may try to formulate a nonlocal theory in which, for example, the electron could interact with a field that was a finite distance away. Then there would be no trouble in interpreting these results, but, as is well known, there are severe difficulties in the way of doing this. Secondly, we may retain the present local theory and, instead, we may try to give a further new interpretation to the potentials. In other words, we are led to regard [the value of electromagnetic potentials at points of space [or space-time] as a physical variable. This means that we must be able to define the difference between two quantum states that differ only by gauge transformation. (pp. 490-491)

They did not mention a possible tertium quid in the form of the postulation of additional field-like entities to restore locality.[30] The first option they offered (nonlocal interaction between the electromagnetic field and the electron) was obviously regarded as a dead end. The second option they apparently regarded as more promising since they promised to show in a future paper how it might be implemented. But rejecting gauge invariance would involve a theory very different from the one they use to formulate the AB effect, and in any case this avenue would also prove to lead to a dead end.

In retrospect it would have been better to adopt a let-the-chips-fall-where-they-may attitude: start with a commitment to gauge invariance, and then ask about the nature of the physical quantities needed for a gauge invariant treatment of quantum electrodynamics; if it turns out that some of these quantities are nonlocal, so be it; try to understand the nature of this nonlocality, its relation to other senses of nonlocality, and how it can be reconciled with the requirements of relativity theory.[31] Strocchi and Wightman (1974) did not think that the chips fell very far; for they conjectured that "the equations of motion for gauge invariant theories can be rewritten in equivalent manifestly gauge invariant form as equations of motion for local quantities" (pp. 2201-2202). Other authors saw the chips fall another way. Thus, Bryce DeWitt (1962) opined that

[30] This option is pursued by Mattingly (2006).

[31] Wallace (2014) argues that the AB effect requires neither a rejection of gauge invariance nor a novel form of nonlocality.

> Nonrelativistic particle mechanics as well as relativistic quantum field theories with an externally imposed electromagnetic field can therefore be formulated solely in terms of field strengths, at the expense, however, of having the field strengths appear nonlocally in line integrals. (p. 2190)

A similar proposal for quantum electrodynamics can be found in Mandelstam (1962). It is unclear, however, how well DeWitt's formulation counts as gauge independent; for it uses a parametrized family of spacetime curves, and the arbitrariness in the choice of such a family amounts to new degree of gauge freedom (see Belinfante 1962). Additionally, Aharonov and Bohm (1962) objected that in DeWitt's formulation there is no essential nonlocality and that the "potentials have been eliminated only in a trivial sense (as we might substitute $y = z^2$ in a linear equation and then assert that the equation is now nonlinear)" (p. 2192).

However, DeWitt's article apparently did have one salutary effect. In a note added in proof he reported:

> The author is happy to acknowledge a stimulating correspondence with Professor Bohm and, although maintaining a different viewpoint, wishes to express his wholehearted agreement with the effort to shift the controversy over the significance of potentials to the arena of local vs. nonlocal theories. (DeWitt 1962, p. 2193)

At this juncture two comments are appropriate, for although they are obvious they seem to have escaped the notice most commentators.

The first comment is that if the key issues that emerge from the debate about the AB effect have to do not with the status of the electromagnetic potentials but with local vs. nonlocal theories then, although Aharonov and Bohm's seminal 1959 article served to force these issues to the forefront of physicists' consciousness, the issues arise quite independently of the AB effect. Forget about the idealizations (F1)-(F3) or (F4)-(F5) needed for the AB effect (requiring that there is a strictly null intersection of the electron configuration space and the region of physical space where the magnetic field is non-zero). Focus instead on real world cases where the electron configuration space is the entirety of physical space, which is itself simply connected. Nevertheless, it must be asked: What is the nature of the observables in a gauge invariant description of quantum electrodynamics? In what sense are they nonlocal? How is this sense of nonlocality related to other forms of nonlocality? Which forms of nonlocality can peacefully coexist with relativity theory and which cannot? Etc. Thus, despite the fact that non-simple connectedness (of the electron configuration space) is essential, by definition, to the AB effect, it is not essential to some of the key issues to which it served to call attention.[32] The philosophical literature seems incapable of absorbing this fact, as if it were under the thrall of the patently invalid inference that goes: 'The AB effect uses non-simple connectedness; the AB effect reveals a _ _ _ _ _ [to be filled in] kind of nonlocality; ergo the nonlocality derives from non-simple connectedness.' To be sure, if actual physical space or the configuration space of an actual electron were non-simply connected, then some form of nonlocality would be in the offing. But this form of nonlocality can tell us nothing about the local vs. nonlocal nature of observables for actual systems since actual physical space and the configuration space of an actual electron are simply connected. Nevertheless, as just noted, gauge-invariant observables for the quantum electrodynamics of actual systems do have a nonlocal character.

The second comment is that the bastardized form of quantum electrodynamics in which the AB effect is usually discussed—an external unquantized electromagnetic field and an electron quantized in non-relativistic QM—is not a felicitous setting in which to try to resolve issues about locality vs. nonlocality. A more appropriate context would be relativistic quantum field theory (QFT). But it is unclear what would constitute an AB effect in terms of fundamental fields. If, as is arguably the case in QFT, particles are not a separate ontological category but are just excitations in the quantum field, then the configuration space of a particle is the configuration space of the field; and the configuration space of the field is the entirety of physical space, which may be assumed to be simply connected.[33] One can imagine quantizing a field on a simply connected space divided by a finite potential barrier into the exterior and the interior of an infinitely long cylindrical region, or the interior and exterior of a toroidal region, and then taking the limit as the barrier becomes infinitely high. Alternatively, one can imagine quantizing the fields on either side of an already existing infinite barrier. If these scenarios make sense in QFT (which remains to be seen), one could then

[32] Recall the discussion of Section 5.3

[33] Of course, mathematically interesting questions can be asked about quantization of fields on a non-simply connected physical space; but this is another matter.

study the relation between the fields on either side of the barrier. It is unclear what would count as a field-theoretic AB effect in such scenarios. It is equally unclear what of importance could be gleaned from such an exercise.

9 Conclusion

Aharonov and Bohm might be faulted for some misdirection in initially focusing attention on the unproductive issue of the reality of the electromagnetic potentials. But hindsight wisdom cannot gainsay their seminal contribution to foundations of QM by forcing physicists and philosophers to confront one aspect of nonlocality in quantum physics. Arguably, this contribution is second only to John Bell's contribution in bringing attention to the nonlocality associated with quantum entanglement. The relation between the two types of locality still a matter for discussion.[34]

The philosophical literature still has not come to grips with some key issues surrounding AB effect. In particular, there is little awareness that the idealizations involved in the AB effect do not comport with the standard accounts of idealizations where the target system is a real world system and where idealizations in the form of simplified/distorted descriptions of the target system are used in an attempt to gain knowledge and understanding this system.[35] This conception of the role of idealization in scientific theorizing would have a hard time explaining why the controversy over the experimental confirmation of the AB effect was so long lasting.

Nor does the philosophical literature show much awareness of the subtleties required to implement the idealizations involved in the AB effect. Some of these subtleties have to do with technical issues about essential self-adjointness of operators. But these technical issues are intertwined with more general methodological issues, of a type all too familiar to philosophers, concerned with trying to answer questions about what would happen under counterfactual scenarios. In particular, it seems that it is hard to say what QM predicts would happen, say, in scattering experiments under the ide-

[34]In this authors opinion, there is no substantial connection between the two types of nonlocality. Bell nonlocality derives from the entanglement of quantum states over local observables whereas the AB effect reveals a nonlocal aspect of quantum observables. But this is a matter for another occasion.

[35]For similar criticisms of standard accounts of idealizations see Shech (2015) and Shech and Gelfert (2016).

alizations required for the AB effect without bringing in additional fictions about how the idealizations are realized.

The literature would also be enriched by devoting more attention to perspective on the AB effect offered by the unitarily inequivalent representations of the CCR and the current algebra that arise when observations are restricted to the electron's configuration space, assumed under the AB idealizations to be disjoint from the flux carrying region. There are (at least) three ways the Stone-von Neumann uniqueness theorem can fail so as to open the way for inequivalent representations. The most widely discussed one occurs for systems with an infinite number of degrees of freedom, as in relativistic QFT (see Ruetsche 2011). The second occurs when the system has a finite number of degrees but the representations are not strongly continuous, as in the polymer representations used in loop quantum gravity (see Ashtekar et al. 2002). The third, discussed here, arises when the configuration space is not simply connected. Comparisons among these three types of cases may help to clarify the physical significance of inequivalent representations.

Despite the fact that the AB effect has been picked over by many able experimentalists, mathematical physicists, and philosophers, it still contains facets and surprises worthy of further exploration. Such inexhaustibility is the mark of truly great discovery.

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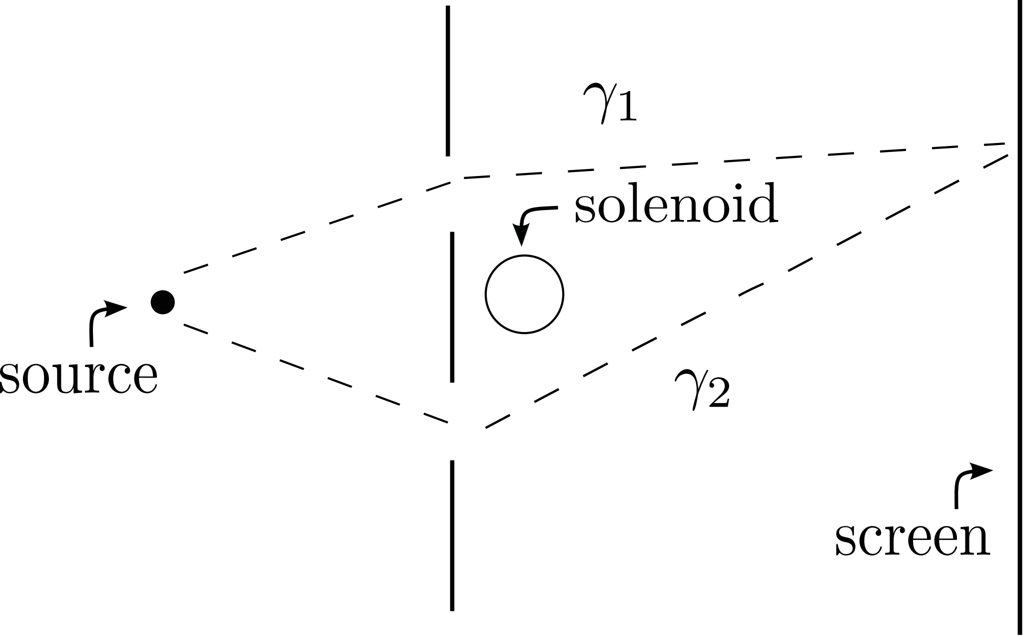

Fig. 1 Interference experiment for the magnetic AB effect

References

[1] Adami, R. and Tata, A. 1998. "On the Aharonov-Bohm Hamiltonian," *Letters in Mathematical Physics* 43: 43-54.

[2] Aharonov, Y. and Bohm, D. 1959. "Significance of Electromagnetic Potentials in Quantum Theory," *Physical Review* 115: 485-491.

[3] Aharonov, Y. and Bohm, D. 1962. "Remarks on the Possibility of Quantum Electrodynamics without Potentials," *Physical Review* 125: 2192-2193.

[4] Arai, A. 1992. "Momentum operators with gauge potentials, local quantization of magnetic flux, and representations of canonical commutation relations," *Journal of Mathematical Physics* 33: 3374-3378.

[5] Ashtekar, A., Lewandowski, J., and Sahlmann, H. 2002. "Polymer and Fock representations for a scalar field," *Classical and Quantum Gravity* 20: L11-L21.

[6] Ballesteros, M. and Weder, R. 2009. "The Aharonov-Bohm effect and Tonomura et al. experiments: Rigorous results," *Journal of Mathematical Physics* 50: 12208-1-54.

[7] Ballesteros, M. and Weder, R. 2011. "Aharonov-Bohm Effect and High-Velocity Estimates of Solutions to the Schrödinger Equation," *Communications in Mathematical Physics* 303: 175-211.

[8] Batterman, R. 2003. "Falling cats, parallel parking, and polarized light," *Studies in History and Philosophy of Modern Physics* 34: 527-557.

[9] Belinfante, F. 1962. "Consequences of the Postulate of a Complete Commuting Set of Observables in Quantum Electrodynamics," *Physical Review* 128: 2832-2837.

[10] Berry, M. 2010. "Aptly named Abaronov-Bohm effect has a classical analog, long history," *Physics Today* 63: 8.

[11] Bocchieri, P. and Loinger, A. 1978. "Nonexistence of the Aharonov-Bohm Effect," *Nuovo Cimento* 47A: 4750-482.

[12] Bocchieri, P. and Loinger, A. 1981a. "Charges in Multiply Connected Spaces," *Nuovo Cimento* 66: 164-172.

[13] Bocchieri, P. and Loinger, A. 1981b. "Comment on the Letter 'On the Aharonov-Bohm Effect' of Boersh et al." *Lettre al Nuovo Cimento* 30: 449-450.

[14] Bocchieri, P. and Loinger, A. 1982. "Remarks on 'Observation of Aharonov-Bohm Effect by Electron Holography," *Lettre al Nuovo Cimento* 35: 370-372.

[15] Bohm, D. and Hiley, B. J. 1979. "On the Aharonov-Bohm Effect," *Nuovo Cimento* 52A: 295-307.

[16] Casati, G. and Guarneri, I. 1979. "Aharonov-Bohm Effect from the 'Hydrodynamical' Viewpoint," *Physical Review Letters* 42: 1579-1581.

[17] Chambers, R. G. 1960. "Shift of an Electron Interference Pattern by Enclosed Magnetic Flux," *Physical Review Letters* 5: 3-5.

[18] de Oliveira, C. R. and Pereira, M. 2008. "Mathematical Justification of Aharonov-Bohm Hamiltonian," *Journal of Statistical Physics* 133: 1175-1184.

[19] de Oliveira, C. R. and Pereira, M. 2010. "Scattering and self-adjoint extensions of the Aharonov-Bohm Hamiltonian," *Journal of Physics A* 43: 354011-1-29.

[20] DeWitt, B. C. 1962. "Quantum Theory without Electromagnetic Potentials," *Physical Review* 125: 2189-219.

[21] Ehrenberg, W. and Siday, R. E. 1949. "The Refractive Index in Electron Optics and the Principles of Dynamics," *Proceeding of the Physical Society* (London) B62: 8-21.

[22] Eskin, G. 2013. "A simple proof of magnetic and electric Aharonov-Bohm effects," *Communications in Mathematical Physic*s 321: 747-767.

[23] Feinberg, E. L. 1963. "On the 'Special Role' of the Electromagnetic Potentials in Quantum Mechanics," *Soviet Physics Uspekhi* 5: 753-760.

[24] Goldin, G. A., Meinkoff, R. and Sharp, D. H. 1981. "Representations of a local current algebra in nonsimply connected spaces and the Aharonov-Bohm effect," *Journal of Mathematical Physics* 22: 1664-1668.

[25] Healey, R. 1997. "Non-Locality and the Aharonov-Bohm Effect," *Philosophy of Science* 64: 18-41.

[26] Healey, R. 2007. *Gauging What's Real. The Conceptual Foundations of Contemporary Gauge Theories*. Oxford: Oxford University Press.

[27] Henneberger, W. C. 1981. "Aharonov-Bohm scattering and the velocity vector," *Journal of Mathematical Physics* 22: 116-117.

[28] Hiley, B. J. 2013 "The Early History of the AB Effect," arXiv: 1304.4736v1.

[29] Klein, U. 1979. "The Inadmissibility of Non-Stokesian Vector Potentials in Quantum Theory," *Lettere al Nuovo Cimento* 25: 33-37.

[30] Kuper, C. G. 1980. "Electromagnetic Potentials in Quantum Mechanics: A Proposed Test of the Aharonov-Bohm Effect," *Physics Letters* 79A: 413-416.

[31] Lyre, H. 2001. "Principles of Gauging," *Philosophy of Science* 68 (Proceedings): S371-381.

[32] Lyre, H. 2009. "Aharonov-Bohm Effect," in D. Greengerger, C. Henschel, and F. Weinert (eds), *Compendium of Quantum Mechanics*, pp. 1-3. Berlin: Springer-Verlag.

[33] Magni, C. and Valz-Gris, F. 1995. "Can elementary quantum mechanics explain the Aharonov-Bohm effect?" *Journal of Mathematical Physics* 36: 177-186.

[34] Mandelstam, S. 1962. "Quantum Electrodynamics without Potentials," *Annals of Physics* 19: 1-24.

[35] Mattingly, J. 2006. "Which gauge matters?" *Studies in History and Philosophy of Modern Physics* 37: 243-262.

[36] Maudlin, T. 1998. "Healey on the Aharonov-Bohm Effect," *Philosophy of Science* 65: 361-368.

[37] McMullin, E. 1985. "Galilean Idealizations," *Studies in History and Philosophy of Science* 16: 247-273.

[38] Nakamura, M. 1990. *Geometry, Topology and Physics*. Bristol: Adam Hilger.

[39] Norton, J. D. 2012. "Approximation and Idealization: Why the Difference Matters," *Philosophy of Science* 79: 207-232.

[40] Nounou, A. M. 2003. "A fourth way to the Aharonov-Bohm effect," in K. Brading and E. Castellani (eds.), *Symmetries in Physics: Philosophical Reflections*, 174-199. Cambridge: Cambridge University Press.

[41] Osakabe, N. et al. 1986. "Experimental confirmation of the Aharonov-Bohm effect using a toroidal magnetic field confined by a superconductor," *Physical Review A* 34: 815-822.

[42] Peskin, M. and Tonomura, A. 1989. *The Aharonov Bohm Effect. Lecture Notes in Physics*, Vol. 340. New York: Springer-Verlag.

[43] Redhead, M. 1980. "Models in Physics," *British Journal for the Philosophy of Science* 31: 145-163.

[44] Reed, M. and Simon, B. 1975. *Methods of Modern Mathematical Physics II*, *Fourier Analysis, Self-Adjointness*. New York: Academic Press.

[45] Reed, M. and Simon, B. 1980. *Methods of Modern Mathematical Physics I*, *Functional Analysis*. New York: Academic Press.

[46] Reeh, H. 1988. "A remark concerning canonical commutation relations," *Journal of Mathematical Physics* 29: 1535-1536.

[47] Roy, S. M. (1980) "Condition for Nonexistence of the Aharonov-Bohm Effect," *Physical Review Letters* 44: 111-114.

[48] Ruetsche, L. 2011. *Interpreting Quantum Theories*. Oxford: Oxford University Press.

[49] Ruijsenaars, S. N. M. 1983. "The Aharonov-Bohm Effect and Scattering Theory," *Annals of Physics* 146: 1-34.

[50] Shech. E. (2015). "Two Approaches to Fractional Statistics in the Quantum Hall Effect: Idealizations and the Curious Case of the Anyon," *Foundations of Physics* 45: 1063-1100.

[51] Shech, E. 2017. "Idealizations, Essential Self-Adjointness, and Minimal Models Explanation in the Aharonov-Bohm Effect," *Synthese* DOI 10.1007/s11229-017-1428-6.

[52] Shech, E. and Gelfert, A. 2016. "The Exploratory Role of Idealizations and Limiting Cases of Models," philsci-archive.pitt.edu/id/preprint13338.

[53] Strocchi, F. and Wightman, A. S. 1974. "Proof of the charge superselection rule in local relativistic quantum field theory," *Journal of Mathematical Physics* 15: 2189-2224.

[54] Sturrock, P. A. and Groves T. R. 2010. "More Variations on Aharonov-Bohm," *Physics Today* 63: 4.

[55] Tassie, L. J. 1963. "The Scattering of Electrons by a Magnetic Field Contained in an Impenetrable Torus," *Physics Letters* 5: 43-44.

[56] Tonomura, A. et al. 1982. "Observation of Aharonov-Bohm Effect by Electron Holography," *Physical Review Letters* 48: 1443-1446.

[57] Tonomura, A. et al. 1986. "Evidence for Aharonov-Bohm Effect with Magnetic Field Completely Shielded from Electron Wave," *Physical Review Letters* 56: 792-795.

[58] Tonomura, A. 2010. "The AB effect and its expanding applications," *Journal of Physics A* 43: 354021-1-13."

[59] Wallace, D. 2014. "Deflating the AB Effect," philsci-archive.pitt.edu/10884/1/.

[60] Wang, R.-F. 2015. "Absence of Electric Aharonov-Bohm Effect due to Induced Charges," *Scientific Reports 5* http://dx.doi.org/10.1038/srep14279.

[61] Weder, R. 2011. "The Electric Aharonov-Bohm Effect," *Journal of Mathematical Physics* 52: 052109.

[62] Weisberg, M. 2007. "Three Kinds of Idealization," *Journal of Philosophy* 58: 639-659.

[63] Weisberg, M. 2013. *Simulations and Similarity: Using Models to Understand the World*. Oxford: Oxford University Press.

[64] Yi, S. W. 2002. "The Nature of Model-based Understanding in Condensed Matter Physics," *Mind and Society* 3: 81-91.