The Sensation and the Stimulus: Psychophysics and the Prehistory of the Marburg School

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Abstract

This paper analyzes the role played by Fechner's psychophysics—the new science meant to measure sensation as a function of the stimulus—in the development of Marburg Neo-Kantianism. It will show how Cohen, in the early 1870s, in order to make sense of Kant's obscure principle of the Anticipations of Perception, resorted to psychophysics' parlance of the relation between stimulus and sensation. By the end of the decade, Cohen's remarks encouraged the early 'Cohen circle' (Stadler, Elsas, Müller) to pursue what were often sophisticated analyses of the problem of the measurability of sensation. This paper argues that in reaction to these contributions, Cohen shifted his interests towards the history of the infinitesimal calculus in his controversial 1883 monograph, *Das Princip der Infinitesimal-Methode*. This book, with its characteristic amalgam of transcendental philosophy and history of science, paved the way to what, around 1900, would become the 'Marburg school' (Natorp, Cassirer, Görland and others). However, it also interrupted a promising discussion in Marburg on the problem of measurability in science.

Keywords: Neo-Kantianism, Marburg School, Hermann Cohen, Psychophysics, Gustav Theodor Fechner, History of Measurement Theory

Introduction

In 1912, Ernst Cassirer (1912) contributed to the special issue of the *Kant-Studien* that honored Hermann Cohen's retirement—his mentor and teacher, and the recognized founding father of the so-called 'Marburg school' of Neo-Kantianism (Poma, [1989] 1997). In the context of an otherwise rather conventional presentation of Cohen's interpretation of Kant, Cassirer made a remark that is initially surprising. It is "anything but accurate," he wrote, to regard Cohen's philosophy as focused "exclusively on the mathematical theory of nature," (Cassirer, 1912, p. 256) as is usually done. A reconstruction of the genesis of Cohen's thought, Cassirer continued, would already refute this interpretation. Actually, "[t]he conditions of the problem, as they were presented to Cohen at that time [die Cohen vorfand], lay at least as much in the critique of *physiology* as in the critique of *physics*" (Cassirer, 1912, p. 256).[1] From the "concept of sensation," Cassirer went on, Cohen was initially led to investigate "the [concept] of 'stimulus'," (Cassirer, 1912, p. 256) which was regarded only in the second instance as a possible object of physics. I suspect that this allusion to the relation between sensation and stimulus, made only in passing, might escape the attention of most readers. However, Cassirer alluded to an over ten year debate initiated by Cohen and continued by a small

[1] Henceforth the letter spacing in the original has been rendered as italics.

group of early sympathizers—a little ‘Cohen circle’ as it might be called, to distinguish it from what only later would become the ‘Marburg school’. Surprisingly, this debate has been completely neglected by historians of Neo-Kantianism, despite being an important line in the development of the history of 19th century philosophy and science.

At the beginning of the 1860s, the nearly sixty-year-old physicist Gustav Theodor Fechner (1860) claimed to have established a new science, which he called ‘psychophysics ’. Psychophysics sought to measure ‘sensation’ on the basis of its functional dependency on the ‘stimulus’, and, at the same time, to present in rigorous form the problem of the relation between the mental and the physical. Starting from Ernst Heinrich Weber’s experimental results (Weber, 1834; Weber, 1846), Fechner suggested that infinitesimal increments of sensation were directly proportional to infinitesimal increments in stimulus and inversely proportional to the amount of the original stimulus. Thus, the function relating sensation to the stimulus would be logarithmic (Fechner, 1860, p. 2:9–14). In the successive decades, a debate was sparked among philosophers, psychologists, physicists, and mathematicians over whether sensations could be measured at all (Heidelberger, [1993] 2004, Michell1999). Many, if not most, of the writings of the members of the ‘Cohen circle’—nearly forgotten figures like August Stadler, Adolf Elsas and Ferdinand August Müller—were meant to contribute to this debate. These sometimes sophisticated analyses were in turn inspired by Cohen’s early attempt to use psychophysics’ conceptual tools to make sense of some of Kant’s obscure remarks about the intensive magnitude of sensation in the *Kritik der reinen Vernunft*.

Michael Heidelberger, in his classic 1993 Fechner monograph (Heidelberger, [1993] 2004, p. Ch. 6), was the first to take the Marburg debate on psychophysics into consideration and to suggest its importance for the formation of the early Cohen circle (Heidelberger, [1993] 2004, p. 124f.). However, Heidelberger’s contribution, which is ‘hidden’ in a monumental investigation of Fechner’s work and its reception, seems to have completely escaped the attention of historians of philosophy. The aim of this paper is to fill what I think is a serious gap in the historical literature on Neo-Kantianism, which has recently been enjoying a renewed interest, especially in the English-speaking world (Beiser, 2014; Makkreel and Luft, 2009; Luft, 2015). Elsewhere (Giovanelli, 2016), I have analyzed in detail the role played by Cohen’s 1883 ‘unsuccessful’ book on the history of the infinitesimal method, *Das Princip der Infinitesimal-Methode* (Cohen, 1883), in shaping some of the fundamental tenets of the school (in particular the intimate relation between transcendental philosophy and the history of science). In the present paper, I aim to show that it was the early Marburg debate on psychophysics that, in a sort of heterogony of ends, prompted Cohen’s interest in the history of higher analysis. This paper is of course very much indebted to Heidelberger’s path-breaking contributions. However, by considering the Marburg debate on psychophysics from the perspective of the history of the Marburg school, rather than of Fechner’s reception, I hope I will throw a different light on the matter. In particular, in my opinion, there is a missing piece in Heidelberger’s reconstruction of the puzzle: Kant’s principle of the Anticipations of Perception (A, p. 166–176; B, p. 207–218). In my view, this is the key element to understanding the entire debate. The Marburg interest in psychophysics arose from Cohen’s early attempt at providing a psychophysical interpretation of Kant’s principle, and then faded away when Cohen became convinced that this attempt had failed. It was at this point that he ventured himself into a new interpretation exposed in *Das Princip der Infinitesimal-Methode*. It was the stance towards Cohen’s ‘paradigm switch’ that determined the Cohen circle’s internal dynamics, which, as we shall see, was more turbulent than it appears in Heidelberger’s presentation.

The narrative structure of the paper will be as follows. In the early 1870s, Cohen, in order to make sense of Kant’s obscure principle of the Anticipations of Perception, initially resorted to psychophysics’ parlance of the relation between stimulus and sensation (sec. 1). The few remarks that Cohen made on the subject encouraged his early followers (Stadler, Elsas and Müller) to pursue an often technical analysis of Fechner's psychophysics (sec. 2). In turn, Cohen, inspired by these critiques, realized that psychophysics was not the proper framework for understanding Kant's Anticipations of Perception, which, he claimed, should be interpreted against the background of the history of the infinitesimal calculus (sec. 3). On the one hand, Cohen's 'infinitesimal turn' divided the early 'Cohen circle' into those ready to follow his new course and those who were taken aback by his unorthodox approach to the calculus (sec. 4). On the other hand, Cohen's *Das Princip der Infinitesimal-Methode* paved the way for the 'Marburg school's' interest in the historical *fieri* of science (Natorp, 1912b). At the same time, however, this new course abruptly interrupted a promising discussion in Marburg on the issue of 'measurability' in science, a discussion that Cohen himself had somehow unwittingly inspired (sec. 5).

1. Anticipations of Perception and Psychophysics: Cohen and Stadler

In the first edition of *Kants Theorie der Erfahrung* (Cohen, 1871), his first Kant monograph, Cohen dedicates several lines to the principle of Anticipations of Perception, the second of the synthetic principles enumerated by Kant in his *Kritik der reinen Vernunft*. In the second 1787 edition of the *Kritik*, the principle somewhat cryptically attributes an intensive magnitude to the *realitas phaenomenon*—the 'real' that is the object of sensation. In the first 1781 edition the wording was slightly different. Kant attributes an intensive magnitude to the sensation *and* to the real which corresponds to it (A, p. 166). Noticing the difference between these two formulations of the principle, Cohen commented: "What is the real as an object of simple sensation, as an intensive quantity, in antithesis to an extensive?": "it is the *unity of the stimulus in which we objectify sensation*" (Cohen, 1871, p. 215-216). Kant of course did not use the expression 'unity of the stimulus' (Cohen, 1871, p. 217). However, Cohen was convinced that Kant's insistence in the second edition of his *opus magnus* on the 'real' that is the object of sensation, rather than on sensation itself, was motivated by the need to find an objective correlate of sensation: "the intention [. . .] of clarifying the real as a simple unity of the objectified stimulus seems to me to be the reason for modifying this affirmation in the second edition" (Cohen, 1871, p. 216).

The language of the "*physiology of the senses*" (Cohen, 1871, p. 215) used by Cohen might not mean much to today's Kant scholar. In contrast, Cohen's insight was enormously influential among his early acolytes, a point that is rarely mentioned in the literature. Cohen's readers at the time probably immediately understood that Cohen's parlance was a reference to Fechner's (1860) controversial attempt to measure the intensive magnitude of sensation in terms of its functional relation to the stimulus. Similar wordings can be found in Cohen's previous 'Herbartian' writings (Cohen, 1868, p. 420f.). However, Cohen might have learned about this issue from Friedrich Albert Lange's *Geschichte des Materialismus* (Lange, 1866). Lange, by discussing the relation between form and content in Kant's philosophy, mentions Fechner's logarithmic formula for the relation between the sensation, as the internal content of consciousness, and the "external (physical) stimulus" (Lange, 1866, p. 251f.).

From 1870 to 1872 Lange was professor of inductive philosophy in Zurich. It was one of his Zurich students, August Stadler (1850–1910), who took up Cohen's insights a few years later. Lange himself had recommended Stadler to Cohen, the latter of which being an unknown *Privatdozent* in Berlin at the time (cf. Cohen, 1910a). In Berlin, Stadler broadened his scientific outlook by following the lectures of some of the great scientists of his time: Hermann von Helmholtz, Emil Du Bois Reymond and Ernst Engel; in addition, he attended Cohen's small seminar on the *Kritik der reinen Vernunft*

(Cohen to Lange, May 14, 1873; Lange, 1968, p. 372). This seminar deeply influenced him. At the end of 1873, Stadler finished writing a short but influential monograph on Kant's teleology (Stadler, 1874), and by October 1875, had already finished a second book, in which he developed Cohen's insight on the transcendental meaning of the *a priori* (Stadler, 1876). Cohen, who in the meantime had succeeded the prematurely deceased Lange in Marburg, immediately elaborated on Stadler's contributions in his 1876/77 book on Kant's ethics (Cohen, 1877).

In his 1876 book, Stadler took an original stance towards Kant's Anticipations of Perception. Stadler, like Cohen, also read Kant's second principle as a claim about the psychology of sensation. However, Stadler was not convinced of the success of Kant's attempt to deduce *a priori* the continuity of the degree of sensation. Thus, he preferred to reformulate Kant's second principle as what he called the "principle of material connection" (*Prinzip der materiellen Verknüpfung*) (Stadler, 1876, p. Ch. VIII). The principle indicates as an *a priori* condition the weaker claim that "all sensations must possess an intensive magnitude" (Stadler, 1876, p. 65). All sensations must be "over the zero-point of consciousness" (Stadler, 1876, p. 65) (that is there are no negative sensations), otherwise the succession of sensations would be interrupted; as a consequence, the unity and identity of consciousness would be compromised and no objectively valid experience would be possible. "Beyond this," Stadler pointed out, Kant took a further step "that I cannot follow" (Stadler, 1876, p. 145). Kant claimed that the increase and decrease of the intensive magnitude of sensation is continuous. Stadler "tried in vain to find a transcendental reason" for this claim, but he had to conclude that "the *Kritik* leaves it unproven" (Stadler, 1876, p. 145). Whether or not the degree of sensation varies continuously can at most be decided *a posteriori*, "through the investigation of the single sensations" (Stadler, 1876, p. 72). However, Stadler would soon show that empirical investigations about sensations, far from supporting Kant's claim, actually refuted it.

At the time, nearly two decades after the publication of the *Elemente der Psychophysik*, the debate about Fechner's result, that physical and subjective intensity are related by a logarithmic function, was alive as ever. In 1877, Fechner published *In Sachen der Psychophysik* (Fechner, 1877), in which he defended himself from his numerous and often renowned critics: Hermann von Helmholtz (1867), Ernst Mach (1863), Joseph Plateau (1872), Joseph Delbœuf (1873), Ewald Hering (1875), Franz Brentano (1874) and others. Fechner's attempt to measure sensation was based precisely on the assumption that the degree of sensation varies continuously along with the continuous variation of the stimulus. It was this assumption that Stadler wanted to challenge, which, he claimed, Fechner's opponents had not called into question. In February 1878, he finished a brief paper entitled "Über die Ableitung des Psychophysischen Gesetzes," which was published in *Philosophische Monatshefte* in the same year (Stadler, 1878). In contrast to e.g. Delbœuf (1878), Stadler did not want to deny the quantitative aspect of sensation, but rather to challenge the empirical adequacy of Fechner's logarithmic formula. We shall roughly follow Stadler's derivation of the latter. Although Stadler's procedure is similar, though not identical to Fechner's, it remained the model of similar derivations presented by the Marburg group. Throughout this paper, I will follow Stadler (who in turn follows Fechner himself) and indicate the variable corresponding to sensation with γ, and that corresponding to the stimulus with β.

Initially Fechner introduced his formula speculatively in the second Appendix of his *Zend-Avesta* (Fechner, 1851, p. 373–386; cf. Scheerer, 1987). However, later, in his *Elemente der Psychophysik* (Fechner, 1860), he presented it as based on Weber's experimental results, in particular those concerning the sensation of weight and touch (Weber, 1834; Weber, 1846). On the basis of numerous trials on pairwise comparisons of weights, Weber found that subjects do not perceive the absolute difference between them but the ratio of difference to the initial weight. If a one-ounce weight is placed in our hand, we can easily perceive it; however, if two weights of, say, 32 and 33 ounces are compared, we do not perceive the one-ounce difference between them; the ratio 1/33 is too small to be discerned. The same can be said for the difference between eye-estimated lengths, sound pitches, etc. Weber's findings can be summarized in the formula:

$$\frac{\Delta\beta}{\beta} = c \tag{i}$$

The constant c depends on the different senses (touch, hearing, etc). In this formulation of Weber's results, there is no mention of sensation. In order to establish a functional relation between sensation and stimulus, Fechner made a further assumption. He postulated that the 'difference sensation' (*Unterschiedsemempfindung*) or 'contrast sensation' (*Contrastempfindung*)—which arises when the difference of two stimuli becomes 'just noticeable'—is proportional to the 'sensation difference' (*Empfindungsunterschied*), the difference between the two corresponding sensations (Fechner, 1860, p. 2:85). This assumption is nothing but the psychophysical analogon of the definition of equal temperature differences in terms of equal volumes of expansion in the theory of heat (Tannery, 1875a; Tannery, 1875b). To put it more precisely: if the just-noticeable stimulus-increase with respect to the original stimulus is constant $\Delta\beta/\beta = c$, then the corresponding 'difference sensation' is constant $\Delta\gamma = c'$. Setting $c = kc'$, the 'just-noticeable differences' between stimuli (j.n.d.) might be used as a unit of measurement; the number of sensation differences $\Delta\gamma$ between two stimuli is k times the number of j.n.d. $\Delta\beta/\beta$ between them. In this way one can write Weber's experimental findings in the form of a functional relation between stimulus and sensation:

$$\Delta\gamma = k\frac{\Delta\beta}{\beta} \qquad \text{where } \ k = \frac{c'}{c} \tag{ii}$$

Notice that this equation does not appear in Fechner's writings (see below in sec. 4.3). In Stadler's reconstruction, however, Fechner started from this equation and postulated that it is valid for every change of sensation, however small; that is, it is also valid for the so-called 'unconscious sensations' that are caused by a stimulus which is not sufficient to raise them to consciousness (e.g., the increase from 32 to 33 ounces). Fechner appealed to what he called an "*a priori* valid mathematical auxiliary principle [Hülfsprincip]" (Fechner, 1860, p. 40): what is true for finite differences ought to also be true in the limit. Then he substituted the finite increments $\Delta\beta$ and $\Delta\gamma$ with the infinitesimally small increments $d\beta$ and $d\gamma$. The simple relation (ii) between two units of measure turned into an informative differential equation, the so-called *Fundamentalformel*:

$$d\gamma = k\frac{d\beta}{\beta} \tag{iii}$$

The next step was to derive an integral formula containing an expression for the measurement of sensation. This is a matter of more or less elementary differential and integral calculus. One first calculates the indefinite integral of eq. (iii) (i.e., without upper and lower limits):

$$\int d\gamma = \int k\frac{d\beta}{\beta} + \text{Const} \tag{iv}$$

By consulting a table of integrals one can easily find that the integral of a fraction whose numerator is the differential of the denominator is the 'natural' logarithm (that is, a logarithm with base $e = 2.7182\ldots$) of the denominator:

$$\gamma = k \ln \beta + \text{Const} \tag{v}$$

To eliminate the constant of integration one evaluates the integral between definite limits. Fechner chooses as the lower limit $\gamma = 0$, where the sensation begins (that is it becomes conscious) and disappears and the correspondent β_0, that is, the threshold of stimulus below which there is no perception:

$$\int_{\gamma_0}^{\gamma} d\gamma = \int_{\beta_0}^{\beta} k \frac{d\beta}{\beta} \tag{vi}$$

The sensation γ—or more precisely the difference of two sensations $\gamma_0 - \gamma$ which corresponds to the differences of two stimuli $\beta - \beta_0$—can be measured as the definite integral, that is, as a summation of infinitesimally small sensation increments $d\gamma$ which corresponds to the summation of infinitely small stimuli increments $d\beta$. According to eq. (v) this is of course equivalent to $\gamma - \gamma_0 = \ln \beta - \ln \beta_0$. For a well-known logarithmic identity, the difference of the logarithms of two numbers is the logarithm of their ratio. Thus Fechner's final equation is the following:

$$\gamma = k \ln \frac{\beta}{\beta_0} \tag{vii}$$

Sensation γ is not simply the logarithm of the stimulus β, but of the latter expressed in terms of its threshold value β_0, the first unit stimulus, from which the zero point where the sensation begins and disappears (Fechner, 1860, p. 2:13). After the initial value (β_0) and the unit of measurement ($\beta_0 = 1$) have been specified, sensation can then be measured as a cumulation of j.n.d. Thus, Fechner's formula is both a 'law of nature' and a 'measurement formula' at the same time (cf. Heidelberger, [1993] 2004, p. 206; Heidelberger, 1993).

After roughly presenting Fechner's derivation in this way, Stadler pointed to a simple but serious conceptual difficulty that it entails. According to Weber's findings, if one imagines the stimulus gradually increasing from a weight of, say, 32 ounces to twice that, then not all of the infinite possible values between 32 and 64 ounces can be perceived, rather only those for which the eq. (i) or $\Delta\beta = c\beta$ holds. We can perceive the difference between, say, 32 and 42 ounces, but not between 32 and 33 ounces: "*not every $\Delta\beta$ corresponds to a variation of the sensation, rather $\Delta\gamma$ remains zero for all values $\Delta\beta > c \cdot \beta$*" (Stadler, 1878, p. 219). When Fechner, relying on his mathematical auxiliary principle, introduced eq. (iii)—that is, he substituted finite differences $\Delta\beta$ and $\Delta\gamma$ with infinitesimally small ones $d\beta$ and $d\gamma$—he contradicted Weber's results rather than building on them.

Of course this objection is valid only if one equates sensation with conscious sensations, as Stadler explicitly does. In this case one can claim that Weber's experiments have shown that no sensation change $\Delta\gamma$ corresponds to a very small stimulus variation $\Delta\beta = d\beta$: "Weber's law is an empirical law and it is valid only for the real, empirically given sensations, and not so-called unconscious ones" (Stadler, 1878, p. 220). As a consequence, we are not allowed to "represent the reciprocal correlation of the stimulus and of the sensation through a continuous function or a curve" (Stadler, 1878, p. 220). In Fechner's conception, stimulus and sensation are related by the natural exponential function $\gamma = e^{\beta}$. Such a function can be plotted on a Cartesian coordinate system by a smooth curve (looking like half a parabola) which increases dramatically over its domain, since γ increases faster as β increases: equal units on a sensation scale correspond to progressively greater units on an external physical scale. However, according to Weber's findings, if we represent a small variation of the stimulus ($\Delta\beta < c$) on the x-axis, then no variation of the sensation ($\Delta\gamma$) would correspond on the y-axis: "The sensation progression," in Stadler's words, "then has the form of a stair with steps of increasing width" (Stadler, 1878, p. 220).

"The essence of the relation between $\Delta\beta$ and $\Delta\gamma$," as Stadler summarized his critique of psychophysics, "is discontinuity. The logarithmic curves, with which one attempts to represent the psycho-physical law, lack empirical truth" (Stadler, 1878, p. 223). However one might judge this technical result, its philosophical implications seem hard to fathom at first. However, the philosophical intent of Stadler's critique of Fechner became more perspicuous in a paper Stadler finished in June of 1880 and published in the same year in the *Philosophische Monatshefte*, "Das Gesetz der Stetigkeit bei Kant" (Stadler, 1880). Stadler showed that Kant had an ambiguous attitude on the question of the continuity of the intensive magnitude. In the Anticipations of Perception, Kant claims that the intensive magnitude of sensation is continuous only in the weak sense, that between every degree and nothing, one can always think of another arbitrary *possible* smaller degree. Nothing can be said about the continuous increasing or decreasing of the variation of degree, which is an empirical question (B, p. 212–213). However, in the *Beweis* of the second Analogy of Experience, Kant seems to defend the stronger claim, that the intensive magnitude of sensation arises from 0 to a certain degree in a continuous manner, running through all *actual* intermediate degrees (B, p. 255–256).

At first sight, Stadler's paper seems to lack theoretical ambition, and it is based on a detailed textual analysis of Kant's passages on continuity. However, though "only in passing," he does note "that modern psychology has not offered any reason to reshape Kant's concept of the degree of sensation" (Stadler, 1880, p. 585). Modern psychophysics does not permit any *a posteriori* demonstration of what Kant attempted in vain to demonstrate *a priori*. Psychophysics postulates that the process of the emergence of a sensation runs through all intermediate degrees, even if this passage is so rapid that it remains unnoticed. However—and this was the result of Stadler's 1878 paper—this postulate, far from being valid a *a priori* as Fechner claimed, can probably be proven wrong *a posteriori*: "as far as intensity is concerned, in my opinion, psycho-physical research has instead shown the discontinuity in psychical transition in relation to the continuous growth of the stimulus" (Stadler, 1880, p. 585–586). Experience seems to show that the stimulus, e.g., a weight, can be increased to a certain degree without causing any change in the corresponding concious sensation.

Cohen seems to have immediately appreciated Stadler's result. In a letter to Stadler on February 24, 1881 Cohen claimed that he now "fully agreed" with his "'Stetigkeit'" (that is, Stadler, 1880). However, he added: "I have outlined a *formulation of the Anticipation* in which your previous concerns seem to be acknowledged and at the same time eliminated" (Cohen to Stadler, Feb. 24, 1881; Cohen, 2015, p. 128–129; my emphasis). This only recently published letter is central to my account. It shows that a fundamental 'paradigm shift' happened at this point. On the one hand, Cohen recognized that Stadler's objections against Kant's *a priori* deduction of the continuity of the degree of sensation were justified. On the other hand, one can surmise that at that time Cohen probably started to realize that Kant's Anticipations of Perception should be understood from a quite different perspective, outside the framework of psychophysics. We do not have further information on what exactly Cohen had in mind. His next letter to Stadler is dated months later in October and includes the first mention of "our new *Privatdozent* Dr. Natorp"; in particular, Cohen announced the latter's new writing "which is thorough a[nd] clear" (Cohen to Stadler, Oct. 30, 1881; Cohen, 2015, p. 131). Paul Natorp's habilitation thesis on Descartes (later published as Natorp, 1882a) had just been accepted and he had given his inaugural lecture on Leibniz a few days earlier (Natorp, 1881). Natorp's early works in Marburg revealed that Cohen was reorienting the interests of his 'circle' towards the 'prehistory of criticism' and its connection to the history of science (Natorp, 1882b; Natorp, 1882c). In particular, Cohen might have already realized that the Anticipations of Perception should be understood not in the context of the epistemology of empirical psychology, but by investigating the historical roots of Kant's principle in the development of modern mathematics and physics.

2. Elsas, Müller and the Early Cohen Circle in Marburg

That Cohen initially attempted to interpret Kant's second principle against the background of Fechner's psychophysics might surprise today's Kant scholars. However, this approach was part of a vast 'research program' which Cohen put forward just after he succeeded the deceased Lange in Marburg, becoming the first Jewish philosophy full professor at a German university. According to the guideline of the Prussian *Kultusministerium*, the philosophy faculty in Marburg used to offer scientific *Preisaufgaben* (essay prizes) with the intent of supporting students (Sieg, 1994, p. 130f.). The 'call for papers' launched by Cohen for the 1880/1881 prize (cf. Holzhey, 1986a, p. 1:381f.) required the candidate to "[e]xplain Kant's mathematical principles"; the first principle, the Axioms of Intuition with reference to "the new science of space," that is, non-Euclidean geometry; "*the second principle,*" the Anticipations of Perception, "*should be evaluated with respect to the problem of psychophysics*" (cit. in Holzhey, 1986a, p. 1:382; my emphasis).

The recipient of the prize was the physicist Adolf Elsas (1855–1895), who, after his dissertation written under Helmholtz's guidance in Berlin (Elsas, 1881), was working as an assistant at the Marburg physical-mathematical institute. In his referee report, Cohen praised Elsas' secure knowledge of Kant's philosophy (Sieg, 1994, p. 131). Concerning the treatment of Kant's first principle, Cohen appreciated Elsas' ability to grasp the philosophical implications of "the Riemmann-Helmholtz speculations" beyond technicalities; with regard to the second principle, Cohen recognized that Elsas presented "the correct point of view for the appreciation of the psychophysical problem" (cit. in Holzhey, 1986a, p. 23f., fn. 86).

The question of the measurability of psychical magnitudes was hotly debated among philosophers at that time. An influential scholar like the great historian of Greek philosophy (and proto-neo-Kantian) Eduard Zeller, had just published a discussion of the issue in the proceedings of the Prussian Academy of Science (Zeller, 1881). The dissertation of Ferdinand August Müller (1858–1888), of which Cohen was the main supervisor, further testifies that this was one of the main philosophical concerns in Marburg. In his *Gutachten*, Cohen emphasized that, after Stadler's technical objection against Fechner's law, Müller was able to show "that in the very problem of establishing a functional relation between stimulus and sensation there is an epistemological mistake" (cit. in Holzhey, 1986a, p. 1:22f.).

In October 1881, Müller finished transforming his dissertation into the booklet *Das Axiom der Psychophysik* (F. A. Müller, 1882). The title refers to the fact that Müller distinguished the 'axiom of psychophysics', that is, Fechner's claim that there is a functional relationship between stimulus and sensation, and the 'problem of psychophysics ', the search for the particular form that such a functional relationship actually assumes (e.g., logarithmic law, power law, etc.). Instead of attacking Fechner's solution of the problem as Stadler had done, Müller wanted to strike the heart of Fechner's enterprise by questioning the very idea that a functional relationship exists at all.

Müller recognized the importance of Stadler's critique of Fechner's law. "Such a sharp objection," he wrote "would alone be capable of overthrowing Fechner's entire construction of formulas [Formelgebäude]" (F. A. Müller, 1882, p. 23), if the latter were based exclusively on Weber's result, that is, only on the 'method of just-noticeable differences'. However, Müller argued, "[t]his is *not* the case"

(F. A. Müller, 1882, p. 23). Beyond the experiments that concern the just-noticeable differences, one must take into account the 'method of more-than-noticeable differences' or the method of bisection. Müller showed that Delbœuf's (1873) repetition of Joseph Plateau's (1872) experiments on color differences allow dividing an initial interval between two largely different perceived magnitudes of a sensation into equal subintervals. Once equally-appearing intervals are defined, according to Müller, Fechner could introduce the hypothesis expressed by Weber's fraction (ii). Müller argued that if one accepts this hypothesis, as Stadler did, then "the passage to the fundamental formula containing the infinitely small values $d\beta$ and $d\gamma$ is irreproachable" (F. A. Müller, 1882, p. 25).

Müller's defense of Fechner's derivation (see however Heidelberger, [1993] 2004, p. 215f.) was of course meant to strategically shift the attention to a more fundamental question. The shortcomings of psychophysics are not physical-mathematical, but, as Müller put it, 'transcendental'. Müller uses the term 'transcendental' according to the interpretation that Cohen had laid down a few years earlier in the first part of his book on Kant's ethics (Cohen, 1877). As is well known, in Cohen's view, Kant's 'transcendental method' proceeds bottom-up from the 'fact' of the mathematical science of nature as it is historically given in the 'printed books', to the conditions of its possibility (cf. Cohen, 1877, p. 77). The same approach must be applied to psychophysics. Quantitative psychologists assumed as a 'fact' that the psychological attributes which they aspired to measure are quantitative. However, this alleged fact must be transformed into a 'problem', and its possibility must be carefully evaluated.

Müller's considerations, unfortunately, presuppose that the reader has already bought into quite a lot of Kant's philosophy. In particular, not surprisingly, Cohen's early interpretation of the Anticipations of Perception plays a major part in Müller's line of argument. Müller conceded that Kant's two formulations of the principle are confusing to say the least. Does Kant claim that sensation has intensive magnitude, or the real, which is the object of sensation, or both? "The relations between sensation and the real, which corresponds to it in the object, is not expressed with full clarity" (F. A. Müller, 1882, p. 51). In Müller's view, Cohen had made a fundamental step toward solving the riddle: "by defining the real that corresponds to sensation in the object as 'stimulus', Cohen developed in a highly significant way Kant's doctrine of the intensive magnitudes" (F. A. Müller, 1882, p. 55). On the one hand, Cohen obtained a hermeneutic elucidation of Kant's different formulation of the Anticipations of Perception in the two editions of the *Kritik der reinen Vernunft*; on the other hand, he provided an epistemological clarification of the ambiguous notion of 'stimulus'. The stimulus does not arouse or cause sensation, the stimulus is the object of sensation or it is the objectified sensation (F. A. Müller, 1882, p. 53). The main consequence of Cohen's approach, in Müller's view, is that it is not the sensation which has intensive magnitude, nor the sensation and the stimulus, but only the stimulus.

According to Müller, "physics measures intensive magnitudes and it is therefore the task of physics to measure the magnitude of the stimulus" (F. A. Müller, 1882, p. 55). Using a Bunsen's grease-spot photometer, for instance, one can establish that the illuminance of the photometer screen due to the source S located a distance d from the photometer is equal to the illuminance of the screen due to the source S' located a distance d' from the screen when the grease spot on the photometer's screen becomes invisible ($S : d^2 = S' : d'^2$). After choosing a luminous intensity of a standard candle (*Normalkerze*) as a unit, it is possible to construct the luminous intensity scale with equally spaced units along the scale. Then one can establish "how many standard candles at the same distance one would need to obtain the *same effect* as the light that we want to measure" (F. A. Müller, 1882, p. 54). Using a Kantian terminology (B, p. 201f., fn.), Müller claimed that intensive magnitudes can be measured through a 'coalition' of parts, rather than through 'aggregation' as in the case of extensive magnitudes (F. A. Müller, 1882, p. 54f.).

In Müller's reconstruction, Fechner attempted to achieve something analogous for the intensity of sensation. As we have mentioned, the trick was to postulate that the 'difference sensation' or 'contrast sensation' (*Contrastempfindung*) for two stimuli (the just-noticeable relative increase in stimulus), was proportional to the 'sensation difference' (*Empfindungsunterschied*) (the difference between the two corresponding sensations). However, Müller objected, this has the "absurd" implication that equal stimuli would produce no sensation (F. A. Müller, 1882, p. 106); but even if one sets aside this issue, there is still no proof that the proportionality postulated by Fechner holds (F. A. Müller, 1882, p. 18). However, without this assumption sensation intensities cannot be transformed into a class of measurable intensities. Instead of speaking of 'contrast sensations' which vary according to their *intensity*, Müller concluded, we can at most speak of 'contrast feelings' (*Contrastgefühle*) that vary according to their *character* (F. A. Müller, 1882, p. IV, 104, 106). In other terms, the equality and difference among, say, luminosities can be organized only in a nominal scale assigning different names to different types of 'contrast feelings' (say, dark, shadowy, bright, luminescent, etc.; cf. Philippi, 1883, p. 585ff. see F. A. Müller, 1882, p. 106–115, for more sophisticated examples).

Thus, contrary to Fechner's ambitions, Müller believed himself to have shown that "sensation cannot be expressed in numbers at all," i.e., it cannot be measured (F. A. Müller, 1882, p. 58). The stimulus can be measured, but one can speak of the magnitude of sensation only in a figurative sense. As a consequence, no functional relationship can be established between them. Fechner's indirect scaling method, in which the sensation differences are measured by putting j.n.d in a row between stimuli becomes powerless (F. A. Müller, 1882, p. 58). The conclusion is of course that the axiom of psychophysics, the very claim that a functional relationship exists between the magnitude of sensation and the magnitude of the stimulus, is flawed: "sensation is not a function of the stimulus, but the stimulus is the object of sensation" (F. A. Müller, 1882, p. 56).

Müller's epistemological reflections constitute only a relatively small portion of the book. The latter includes further detailed analyses of Weber's and Charles Delezenne's (1826) experiments (II.1 and II.2), of Helmholtz's (1877) analysis of sound sensations (II.3–5), of Georg Elias Müller's (1878) development of the right or wrong cases method (III.1), Delbœuf's (1873) measurement of fatigue (III.2), and Hering's (1875) work on spatial and temporal sensations (III.1). This material cannot be considered here. What is relevant in this context is that Müller displayed a solid technical knowledge of the topic. Thus, Fechner himself took the time to reply to the "mathematically as well as philosophically trained author" (Fechner, 1882, p. 324).

In 1882, Fechner published *Revision der Hauptpuncte der Psychophysik* (Fechner, 1882), a newly articulated defense of psychophysics against an apparently never ending series of new critics, including several philosophers like Zeller (1881, 1882) and Johannes von Kries (1882). If Zeller insisted that sensation magnitudes cannot be measured in practice, von Kries thought that they cannot be measured in principle; Müller, as we have seen, raised the more radical 'quantity objection' that there are no sensation magnitudes at all (cf. eg. Michell, 1999, p. 40ff.). Against Müller's argument "from Kantian principles" (Fechner, 1882, p. 325), Fechner pointed out that no one can deny that sensations of the same type (light, acoustic, etc.) can be said to become stronger or weaker. Müller can regard 'contrast sensations' or sensation differences as mere 'contrast feelings', if he wants to. However, Fechner believed himself to have shown that if one *defines* 'sensation differences' as proportional to "difference sensations," one can achieve measurements that are empirically correct.

"What should I care about Kantian definitions" (Fechner, 1882, p. 325), he concluded.[2] Müller is correct in claiming that quantities can be compared only via a unit of measure: "and exactly in this way my measurement formula measures sensation, even if not directly, but through the mediation of its functional relations with the stimuli" (Fechner, 1882, p. 326).

In his review of Müller's book, Elsas (1883a)—who in the meantime had become *Privatdozent* in Marburg (Elsas, 1882)—defends his Marburg colleague on this point. If sensation intensities are ordinal (weaker and stronger), this does not mean that they are measurable, that a certain sensation is five or six time stronger than another one. Neither this measurability of a quantity can be inferred from the fact that it is set in functional relation to a measurable quantity. For instance, the welfare of a nation depends, say, on its morality (*Sittlichkeit*); if one concedes that the former is measurable, this does not mean that the latter is too (Elsas, 1883a, p. 130-131). Fechner's objection to Müller, according to Elsas, reveals that the problem was much deeper. Fechner and his followers should undertake a serious discussion to establish what a 'measurable magnitude' is in general (Elsas, 1883a, p. 131). Physics could be successful for a long time without raising this question, but empirical psychology made an epistemological analysis of the issue unavoidable.

Elsas's review is worth mentioning because it reveals the background against which this issue was understood within the 'Cohen circle'. According to Elsas—who was probably summarizing the results of his prize essay—Kant's transcendental question about the possibility of mathematics and physics should be extended to the new sciences that were gaining ground in the second half of the 19th century. "Is metageometry, is psychophysics a possible science?" (Elsas, 1883a, p. 127). The issue, as Cohen has shown, is not a physio-psychological one; the origin of the representation of space or the organization of our sensibility are not at stake. The question is, "on which transcendental foundations (that is, on which conditions making the knowledge possible) is the necessity of mathematical knowledge based?" Müller, embracing "Cohen's conception of the transcendental" (Elsas, 1883a, p. 127), has ventured to submit psychophysics to such a critical investigation.

In mathematics and physics we establish functional relationships among magnitudes. The stimulus is clearly a magnitude that can be measured. "Can the sensation also be measured? Yes or no? The answer to *this* simple question decides the possibility of psychophysics" (Elsas, 1883a, p. 130). Despite providing an overall positive review of the book, Elsas however did not fully agree with Müller's philosophical conclusions. Elsas, like Müller, subscribed to Cohen's identification of Kant's 'real which corresponds to sensation' with the 'stimulus'. However, Elsas denied that one can attribute an intensive magnitude to the stimulus: "Physics measures *intensities* only as extensive magnitudes; the intensity of a *physical* phenomenon, e.g. of sound, of a light source, of a force is never an intensive magnitude" (Elsas, 1883a, p. 133). Intensive magnitudes are measurable only indirectly through their extensive effects.[3]

3. Cohen: From Psychophysics to the History of the Infinitesimal Method

When Elsas published his review of Müller's book, Cohen had already come to the conclusion that his interpretation of the Anticipations of Perception, in which the 'real' was identified with the stimulus of psychophysics, was not satisfying. Psychophysics was simply not the right framework for making sense Kant's Anticipations of Perception. As we have mentioned, at the beginning

[2]For the importance of Fechner's answer to Müller, cf. Heidelberger, [1993] 2004, p. 240.
[3]This is, by the way, a quite Kantian point of view, see, e.g. Ak., p. 18:322; 28:425.

of 1881, Cohen wrote to Stadler that he envisaged a way to acknowledge, and at the same time overcome, the latter's objection that, contrary to Kant's claim, the degree of sensation probably varies discontinuously. The most reliable, though indirect, source at our disposal for concretely understanding what Cohen had in mind is probably Stadler's then-new monograph, *Kants Theorie der Materie* (Stadler, 1883)—possibly the first monograph on Kant's *Metaphysische Anfangsgründe der Naturwissenschaft* (Kant, 1786).

Stadler still fully works within the framework built by Cohen and discussed by the early Cohen circle. As Stadler writes, "Cohen, very happily, called 'stimulus' the objective correlate of the intensive magnitude" (Stadler, 1883, p. 60). The magnitude that corresponds to the stimulus of the intensive magnitude of sensation would thus be called the "magnitude of the stimulus" (Stadler, 1883, p. 60). Stadler agreed with Müller that "only the stimulus can be measured, not sensation"; however, this did not mean that "intensive magnitude can only be attributed to the stimulus," as Müller claimed (Stadler, 1883, p. 248, n. 24). For Stadler, only sensation has intensive magnitude, but this is an internal psychological determination, which "in its own nature is not measurable" (Stadler, 1883, p. 61). Stadler agreed with Elsas that intensive magnitudes are measurable only through their causal product of extensive ones.

Stadler attempted to apply these conceptual tools to Kant's work, giving of a sort of psychophysical interpretation of some of the key elements of his philosophy. Kant's *Gegebenwerden* and the *Afficirtwerden* can be interpreted as, in a first approximation, the "representation of the dependence of the change of sensation on the outside" (Stadler, 1883, p. 53); more precisely this dependence—"which today one would call psychophysical" (Stadler, 1883, p. 58)—means "the emergence of a degree of consciousness, an intensive magnitude" (Stadler, 1883, p. 59). The objective correlate of the intensive magnitude, as suggested by Cohen, is the 'stimulus'. In Stadler's view, the stimulus of sensation should ultimately be thought of as motion. This is what Kant meant when he claimed that the object of the external senses must be motion "because only thereby [through motion] can these senses be affected" (Ak., p. 4:477). The differences between sense qualities should be ultimately dissolved in the differences between motions. In Stadler's reading, this is nothing but "the principle of physiology that all external stimuli of the sensations must be motions" (Stadler, 1883, p. 8), which, through the peripheral nervous system are passed to the central nervous system (cf. Stadler, 1878, p. 223; see also Wundt, 1874, p. 277).

Stadler notices further that, in the *Beweis* of the Anticipations of Perception, Kant used the expression 'moment' to indicate the "reality as cause" and, in particular, as the "cause of sensation," as something that exerts an influence on the senses (B, p. 211). 'Moment' is for Kant the 'moment of acceleration,' an infinitely small variation of velocity. According to Stadler, the term 'moment' reveals that what Kant called "'influence on the senses', the 'stimulus'" (Stadler, 1883, p. 60) is nothing but the effect on what in physics we call 'force'. The "*moment* is the magnitude of the force that corresponds to the intensive magnitude of the sensation" (Stadler, 1883, p. 60). The magnitude of the force and intensive magnitude are correlated, but not identical. Force can be measured only through its extensive effects. The intensive magnitude is given in consciousness, it is a "subjective evaluation" of the stimulus, and cannot be measured (Stadler, 1883, p. 61).

Stadler conceded that there are passages where Kant seems to attribute intensive magnitude not to sensation, but to physical determinations like 'velocity' (cf. Ak., p. 4:540–541). However, he claimed, one should resist confusing this intensive magnitude with the intensive magnitude of sensation. According to Stadler, Kant "did not want to identify it [the intensive magnitude of velocity] at all with the intensive magnitude, which corresponds to reality" (Stadler, 1883, p. 37; in the sense of the category of reality). The definition of the velocity as an intensive magnitude, Stadler pointed out, was only an analogy used to emphasize that the magnitude of velocity is not composed of parts, as the magnitude of space and time is.

Stadler took some pains to interpret away the passages that could support the opposite reading. E.g., he comments on Kant's reflection with the title 'Über das Moment der Geschwindigkeit im Anfangsaugenblick des Falls' (Ak., p. 14: Refl. 67; 1788-1791). Here Kant attributes an 'intensive magnitude' to the 'moment of velocity'—that is, the tendency to fall downward at the beginning of a falling motion—and conceives of the finite motion as a summation of infinitely many 'moments' (Ak., p. 14:495; Refl. 67). Stadler warned that one should not try to read passages like these as Kant's attempt to provide a foundation for the "objective validity of the differential calculus" (Stadler, 1883, p. 39). Interpreters committed to such a reading "would be in contradiction with the view that Kant expressed of the infinitesimal method" (Stadler, 1883, p. 39). In general, in Kant's work, continuity means infinite possibility of division, not composition from actual, though infinitely small parts. Stadler concludes polemically that, "those who make the intensive magnitude correspond with the differential confuse the form with the content" (Stadler, 1883, p. 39).

Although Stadler never mentioned Cohen explicitly, Cohen himself later read this last claim in particular as being directed towards his upcoming book (Cohen, 1910a). Stadler's monograph was finished in March (Stadler, 1883, p. IV). According to Cohen's later recollections, Stadler stayed in Marburg in the summer of 1883 while Cohen was working on his *Das Princip der Infinitesimal-Methode* (Cohen, 1883). The *Vorwort* of Cohen's book is dated August 1883 (Cohen, 1883), but the book was sent to print only in mid-October (Cohen to Natorp, Sep. 27, 1883; CN, p. Br. 1, 148). Reading the drafts of their books, Stadler and Cohen probably realized that one of them put forward precisely what the other vehemently rejected. By that time, Cohen had completely abandoned the framework of psychophysics, which had enjoyed so much success among his acolytes. He became convinced that Kant's second principle could be understood precisely by looking at the connection between the concepts of 'moment', 'intensive magnitude' and 'reality', which was suggested by the Kant passages mentioned by Stadler. According to Cohen, by establishing this connection, Kant had expressed philosophically in his principle of Anticipations of Perception the problem that Galilei, Leibniz and Newton had tried to answer mathematically when they introduced a new type of quantities, namely, 'infinitesimally small quantities'.

I cannot enter into the details here of this highly obscure book and its tormented reception, which I have described in detail elsewhere (Giovanelli, 2016). What I would like to emphasize is that Cohen explicitly recognized that it was Stadler's critique of a possible psychophysical reading of Kant's Anticipations of Perception that changed his mind: to understand "what was new and valuable in Kant's conception of the intensive magnitude," Cohen wrote, it was necessary to become aware of "the deficiencies in its foundation and presentation, which A. Stadler had first emphasized" (Cohen, 1883, p. 105). Stadler showed the failure of psychophysics' attempt to present the intensive magnitude of sensation as the differential $d\gamma$ (Cohen, 1883, p. 159–160).[4] Cohen, however, at the turn of the 1880s, must have come to realize that what psychophysics had failed to achieve in the case of sensation could still be established on another basis. The intensive magnitude of sensation was not at stake in Kant's Anticipations of Perception, but rather the intensive magnitude of physical determinations such as 'velocity'. It was in the attempt to give a mathematical counterpart to the intensive magnitude of velocity that the 'differential' was introduced. Kant's principle was the

[4]See cf. Heidelberger, [1993] 2004, p. 222f. for more on Cohen's critique of psychophysics in *Das Princip der Infinitesimal-Methode*. In my perspective, the interesting part of the story is the role psychophysics played in Cohen's work *before* this book was written.

philosophical expression of this historical fact.

The passage from one interpretative framework to the other seems to be based more on an association of ideas than on a proper argument. However, the role played by the discussion of psychophysics is confirmed by Cohen's own reconstruction of the path that led him to rethink the interpretation of Kant's second principle in the revised and greatly augmented edition of his *Kants Theorie der Erfahrung* (Cohen, 1885), which was finished in August 1885. Cohen recalls that "[i]n the first edition of this book" (cf. Cohen, 1871, p. 46), he had attempted to identify the 'real' which corresponds to sensation with "the 'unity of stimulus, in which we objectify sensation'" (Cohen, 1885, p. 436). However, he now recognized that "this expression, although useful to encourage further reflections, was too psychological to be maintained" (Cohen, 1885, p. 436). What is at stake is not "the unit of stimulus", but the "the unit of motion" (Cohen, 1885, p. 436). It is precisely in those passages mentioned by Stadler that Kant, by establishing a relation between reality, intensive magnitude, and the mathematical/physical concept of 'moment', expresses the very problem that the 'discoverers' of the infinitesimal calculus had to solve to make motion an object of scientific inquiry: "Consequently Galilei and Leibniz talk of the infinitely small as an intensive magnitude" (Cohen, 1885, p. 427). Kant, Cohen claims, does not even have to emphasize this connection since it was obvious to his readers (Cohen, 1883, p. 105).

Again Cohen recognized that it was Stadler's work on psychophysics that led him to completely rethink his own interpretation of the Anticipations of Perception. In Cohen's words, Stadler "rightly opposes the continuity of sensation as an *a priori* determination" (Cohen, 1885, p. 437). Not only is there no 'pure transcendental foundation' for the continuity of sensation; Stadler also demonstrated that empirical studies on the psychology of sensation are even less capable of demonstrating *a posteriori* that sensation grows in a continuous way. However, Stadler "only looks for the pure transcendental foundation he is missing in the sensation" (Cohen, 1885, p. 437). He did not realize that "the 'pure transcendental foundation'" he was looking for cannot be found "in the sensation, but only *for* the sensation" (Cohen, 1885, p. 437; my emphasis). That transcendental ground should be found "in the new and autonomous mode of magnitude, which in the infinitesimal method reveals itself to be fruitful for the constitution of the object in his mechanical meaning" (Cohen, 1885, p. 437). In other words, Stadler still thought that psychophysics offered the conceptual framework for understanding the problem Kant was facing. According to Cohen, however, this framework was inadequate for understanding Kant's concept of the "intensive": "the so-called *intensity of sensation must absolutely be distinct from the intensive magnitude or reality of sensation*" (Cohen, 1883, p. 156).

Elsas, in his 1885 review of Stadler's 1883 book (Elsas, 1885b), points out clearly where Stadler was no longer in accordance with Cohen's new approach to Kant's second principle. Stadler considered "motion as the stimulus of sensation" (Elsas, 1885b, p. 146); according to Elsas, however—even if there is some textual evidence to apply this psychological interpretation to Kant—the central point is different: transcendental philosophy should try to explain what makes general mechanics a science and thus what makes 'motion' a legitimate object of scientific inquiry: "The task is to explain the fact of science, the fact of general mechanics" (Elsas, 1885b, p. 146). The relation between motion and sensation is philosophically secondary, if not irrelevant. "Maybe the author should have further developed the concept of intensive magnitude and its relation to the category of reality" (Elsas, 1885b, p. 146); "the elementary concepts of mechanics," Elsas continues alluding to Cohen's work, "already offer some indication in this direction" (Elsas, 1885b, p. 146).

4. The Dissolution of the Cohen Circle

Stadler's publication rate fell drastically in the ensuing years, possibly for health reasons, and we do not have textual evidence of his possible counter-objections. However, we know that he never agreed with Cohen's new course, even if he himself unwittingly had brought it about; their friendship, however, never suffered (Cohen, 1910a). Elsas continued to publish, as a physicist (Elsas, 1883b; Elsas, 1885a; Elsas, 1886c; Elsas, 1887), a reviewer of philosophical publications (Elsas, 1884; Elsas, 1885b) and as a high quality science popularizer (Elsas, 1886a). Moreover, after Fechner's 1882 *Revision der Hauptpuncte der Psychophysik* (Fechner, 1882), Elsas returned to psychophysics; in 1886 he published his philosophical reflections on the topic in a little pamphlet, *Über die Psychophysik* (Elsas, 1886b). Elsas continued the early Marburg debate on the 'measurability of sensation' and at the same time made some timid attempts to integrate it into the new approach taken by Cohen in his 1883 monograph.

4.1. Elsas' Über die Psychophysik

Elsas organized his book around two questions: 1. "Are Fechner's measuring formulas mathematically and physically correct and derived from the data?" (Elsas, 1886b, p. VI). 2. "Is psychophysics in the sense of Fechner possible in general?" (Elsas, 1886b, p. VI). Elsas answered both these questions with a resounding 'no'. In this sense Elsas was more radical than both Stadler and Müller. In the language introduced by the latter, he intended to criticize both the 'problem' and 'axiom' of psychophysics. In what follows, for reasons of brevity I will concentrate on the third and final part of the book, in which Elsas dealt with question 2., which is of course the one richer in philosophical implications (see also Heidelberger, [1993] 2004, p. 229ff.).

After a long digression on Bois-Reymond's (1882) theory of quantity (Elsas, 1886b, p. 50–61; cf. Darrigol, 2003, p. 540)—Elsas argued against the very possibility of 'measuring the sensation' via its functional relation with the stimulus. Elsas defined the concept of 'function' as the 'production' of quantity from other quantities. For instance, the area of a triangle is a function of two variables, base and altitude (Elsas, 1886b, p. 61). However, something different is meant when one says that, according to Ohm's law, the current through a conductor between two points is a function of the voltage across the two points and a constant of proportionality, the resistance (Bois-Reymond, 1882). According to Elsas, in the latter case we assume, at least implicitly, that there is a *causal connection* that serves as the basis of the functional relationship (Elsas, 1886b, p. 61f. cf. Heidelberger, 2010). The volume of *Knallgas* or oxyhydrogen that can be produced by water electrolysis in a given amount of time *measures* the electric current because the current causes the electrolysis; or, to take a simpler example, the thermometer *measures* temperature because equal temperature differences *cause* equal expansions of the mercury column, etc.

No such causal connection can be found between stimulus and sensation. Thus, Fechner has no real reason to claim that equal 'difference sensations' are proportional to equal 'sensation differences'. The question is not merely epistemological; Elsas showed that, without having a causal connection as a guide, it is easy to lose grasp of the mathematical derivation. Let's grant Fechner that his "constant k has the same meaning as the practical measuring unit for sensation" (Elsas, 1886b, p. 17); then inconsistencies arise in his system of formulae. According to eq. (ii), k is the value of the sensation difference $\Delta\gamma$ that corresponds to $\Delta\beta : \beta = 1$, that is, to a doubling of the initial stimulus β; according to eq. (v), k is the value of the sensation difference $\Delta\gamma$ that corresponds to $\beta = e$, where e is the base of the natural logarithm (the natural logarithm of e is 1); according to eq. (vii), $\Delta\gamma = k$ when $\beta_0 : \beta = 1 : e$ (the natural logarithm of 1 is 0).[5] Thus, without an underlying causal connection, a mere functional relation is not sufficient to establish a quantitative comparison.

It is true that, as Fechner points out, even in physics a functional relationship does not always imply a causal relationship (Fechner, 1882, p. 227); the frequency of a pendulum is a function of the pendulum's length, the orbital velocity of a planet of its distance from the sun, etc.; however, no causal relationship between these pair of quantities is implied (Elsas, 1886b, p. 65). However, according to Elsas, at a deeper level, there must always be "a causal condition at the basis of the functional connection" (Elsas, 1886b, p. 65); in the cases just mentioned, it is the gravitational force (cf. Heidelberger, [1993] 2004, p. 230ff.). Thus there are only two alternatives. If, as Fechner claims, there is no causal relationship between the body and the mind, then the connection between stimulus and sensation cannot be expressed through a mathematical equation; the latter is not measurable. What Fechner constructed is only "pseudo-physics [Scheinphysik]" (Elsas, 1886b, p. 67). If Weber's law were really an expression of the causal connection between stimulus and sensation, then Fechner's psychophysics would be a "real physics of the soul" (Elsas, 1886b, p. 64). However, the existence of a causal relationship between stimulus and sensation, the body and soul, seems to be incompatible with energy conservation.

Thus, Elsas did not seem to allow for any appeal against his draconian statement: "Mathematical psychology, psychophysics and physiological psychology—three absurd expressions [Bezeichnungen]!" (Elsas, 1886b, p. 79). The only escape, in Elsas' view—as Stadler had already suggested (Stadler, 1878, p. 223; cf. Elsas, 1886b, p. 74; fn. 20)—is to consider the causal relationship not between stimulus and sensation, but between stimulus and the peripheral/central nervous system: "then psychophysics becomes nothing but common physiology" (Elsas, 1886b). As Elsas put it in the opening of a contemporary semi-popular introduction to acoustics, "[t]he object of sensations is always motion, which is transmitted to the nerves of our sense organs and through them is transported to our brain" (Elsas, 1886a, p. 1). We can say that hearing is different from sight, but we are not able to express in words this difference between sound and color. The only difference is in the types of motion that excite our sense organs; this difference is investigated by physics, "the task of which is to reduce the natural phenomena to motion" (Elsas, 1886a, p. 1). If one cannot make use of the concepts of force and motion, Elsas concludes rather narrow-mindedly, then no mathematics can be applied (Elsas, 1886b, p. 70). Sensation is not an object of scientific knowledge; it is not a part of nature; it has no reality for the mathematical physicist; it cannot be treated mathematically as a measurable quantity (Elsas, 1886b, p. 70).

4.2. With or Against Cohen

The role of sensation, Elsas concluded in his book, is epistemological. Sensation expresses the need to go beyond measurable extended quantities, to look at that 'something' which is extended, and provides physical content to extension. This is, in Cohen's interpretation, what Kant had tried to express in the Anticipations of Perception, by connecting sensation, reality and intensive magnitude (Elsas, 1886b, p. 75f.). E.g., as Elsas put it, 'velocity' as a physical concept cannot be reduced to the differential quotient of space and time: it "is an intensive magnitude; the extensive quotient is only the mathematical expression of it but not an adequate one" (Elsas, 1886b, p. 68). In this way Elsas attempted to 'rephrase' Cohen's infamously obscure prose in a language more familiar to science

[5] Recall that eq. (vii) is derived from $\gamma_0 - \gamma = k \log \beta_0 - \log \beta$.

practitioners: “At least I hope that I have not changed something essential in this translation from the scholastic language of the philosophers into the one of the physicists” (Elsas, 1886b, p. 76).

These remarks seem to be tacked on at the end of the book, rather than part of its main line of argument.[6] However, they show that, in contrast to Stadler, Elsas had at least tried to embrace Cohen’s new course. This was far from obvious. In contrast, Müller’s attitude towards Cohen’s work dramatically changed after the latter’s 1883 book. In his 1886 habilitation writing, *Das Problem der Continuität im Mathematik und Mechanik* (F. A. Müller, 1886), Müller became highly critical of Cohen’s connection of the ‘differential’ and the intensive magnitude. Interestingly, according to Müller, the “difficulties which Cohen got tangled up in the development of his thought” should be traced to the fact that he was misled by the analogy with psychophysics. Cohen, in Müller’s view, “committed the same mistake that psycho-physics fell into” (F. A. Müller, 1886, p. 96n.); he attributed “to something which, like the differential, is not a real object [...], a magnitude, indeed an intensive magnitude” (F. A. Müller, 1886, p. 96n.). Müller’s remark is important insofar as it confirms that Cohen, more or less consciously, tried to transfer the approach that psychophysics had applied to the intensive magnitude of psychological quantities to the intensive magnitude of physical ones. Just as psychophysics wanted to measure sensation as the accumulation of infinitesimally small sensation increments, Cohen wanted to present the ‘production’ of finite quantities in physics as the infinite summation of infinitesimal quantities. However, as Müller pointed out, this is “a complete misunderstanding of the method of limits” (F. A. Müller, 1886, p. 96n.).

Müller’s attacks, who “for some years was part of our restricted circle,” came as a surprise (Elsas to Lasswitz, Jan. 7, 1887; CN, p. Br. 11, 171). Elsas pointed this out in a letter to Kurd Lasswitz at the beginning of 1887, when he thanked the latter for the positive review (Lasswitz, 1887a) of his psychophysics booklet in the *Deutsche Literaturzeitung*—Germany’s most important general review journal. Elsas did not want to influence Lasswitz’s judgment, whom Natorp had asked to review Müller’s book for the *Philosophische Monatshefte* (Natorp to Lasswitz, Sep. 24, 1886; CN, p. Br. 10, 170). However, he did not hide his disappointment towards Müller’s behavior, after all Cohen had done for him (Elsas to Lasswitz, Jan. 7, 1887; CN, p. Br. 11, 171f.). Nevertheless, he also had to admit the difficulties of fully embracing Cohen’s approach: “Often I myself am not sure if Cohen really means and says what I read off or hear from his elliptic remarks” (Elsas to Lasswitz, Jan. 7, 1887; CN, p. Br. 11, 172). Elsas expressed a similar uneasiness a month later in another letter to Lasswitz (Elsas to Lasswitz, Feb. 8, 1887; CN, p. Br. 12, 182).

Lasswitz (1848–1910) was a high school teacher from Gotha who was working on the history and philosophy of atomism (Lasswitz, 1878; Lasswitz, 1884). He became interested in Cohen’s work on the history of the infinitesimal calculus (Lasswitz, 1885a; Lasswitz, 1885b), possibly after a correspondence (Eccarius, 1985) with one of Cohen’s most renowned mathematician critics, Georg Cantor (Cantor, 1884). Lasswitz’s review of Müller’s book became a long paper entitled “Das Problem der Continuität” (Lasswitz, 1888). Beside addressing some of Müller’s criticisms (Lasswitz, 1888, p. 24–28; see also Lasswitz, 1887b), Lasswitz mostly gave his take on Cohen’s connection between the differential and intensive magnitudes. Motion, he explained, as a change of space extension in a certain time span, has no physical reality. In every instant, there is no change of position,

[6] I disagree with Heidelberger’s claim that Elsas “associates this [his critique of psychophysics] with Cohen’s theory of infinitesimals” (Heidelberger, [1993] 2004, p. 230). I suspect that Elsas’s emphasis on “role that causality plays in measurement” (Heidelberger, [1993] 2004, p. 229) was formulated before (possibly already in the answer to the 1881 *Preisaufgabe*) and independently from Cohen’s 1883 book; only as an afterthought Elsas tried to give some Cohenian flavor to his line of argument.

thus no motion. The physical reality of motion has to be searched for in something that is beyond extension, but possesses a greater or smaller ability to generate a determinate motion (Lasswitz, 1888, p. 19ff.). However, this 'intensity' of motion finds mathematical expression in the differential *function* $dy = f'(dx)$, and not in the differential dx, as Cohen misleadingly claimed (Lasswitz, 1888, p. 29–31). Yet Müller remained unconvinced. He replied to Lasswitz in October 1887, calling Cohen's work on the infinitesimal method "one of the most monstrous births in the entire history of philosophy" (Müller to Lasswitz, Oct. 26, 1887; CN, p. Br. 13, 172). Müller died a few months later of a severe lung disease, so that no further discussion was possible.

4.3. The Fechner-Elsas Debate

Toward the end of 1887, Elsas sent Lasswitz detailed comments on the latter's "essay on the problem of continuity" (Elsas to Lasswitz, Nov. 9, 1887; CN, p. Br. 14, 179). He also took the opportunity to announce that he had finished another paper on psychophysics for the *Philosophische Monatshefte*: "Even though I'm very busy with accumulators and galvanometers, I did not want to hesitate too long in answering Fechner's paper in the *Philos. Studien*" (Elsas to Lasswitz, Nov. 9, 1887; CN, p. Br. 14, 182). In fact, the indefatigable Fechner again made the effort to answer some of his critics, Elsas himself and Alfred Köhler (1886), in Wilhelm Wundt's journal (Fechner, 1888) which, in spite of the title, mainly dealt with empirical psychology. Fechner was 86 years old, and as he recognized half jokingly, this may have been the last time that he was able to defend his own scientific creature (Fechner, 1888, p. 163).

Fechner's reply to Elsas is too long and detailed to be dealt with here. However, a point is worth noticing. Here Fechner repeated, even more clearly than in Müller's review, that the assumption that 'difference sensations' are proportional to 'sensation differences' was merely the "simplest hypothesis" (Fechner, 1888, p. 171, 174) from which he could choose (cf. also Fechner, 1888, p. 147ff. for more details). Fechner calls it the *Unterschiedshypothese*, the 'difference hypothesis' (j.n.d.s are proportional to sensation differences). Such a hypothesis is acceptable inasmuch as it is implied by the empirically confirmed logarithmic law. Fechner claims that there is at least one other simple alternative one can think of, which he calls the *Verhältnishypothese*, the 'ratio hypothesis' (j.n.d.s are proportional to sensation ratios). This hypothesis, which was adopted by Plateau and Brentano (and supported by Elsas himself), leads however to a power law which, Fechner claimed, is empirically wrong (but see of course Stevens, 1961).

However, Fechner had to concede Elsas' point that, if one assumes the 'difference hypothesis', eqs. (ii), (v) and (vii) are indeed incompatible "as the author [Elsas] has correctly shown on p. 17" (Fechner, 1888, p. 167). However, Fechner had a quite interesting response. He claimed that the proportionality between sensation differences and difference sensations applies only to the differential equation (iii), and not to the finite equation (ii), an equation that he actually never wrote: "where for God's sake did I ever put forward equation [ii]?" (Fechner, 1888, p. 168). Thus, in the reply to Elsas, Fechner may have revealed a quite astonishing fact which, Neo-Kantianism aside, is relevant to Fechner scholarship in general (cf. Scheerer, 1987). Fechner did not regard eq. (iii) as an approximation derived from the empirically verified eq. (ii), but vice versa: he saw eq (ii) as the approximation of the exact eq. (iii) (cf. Dzhafarov and Colonius, 1999). After all, this is confirmed by the fact that, in the *Zend-Avesta*, he had introduced the differential equation as an exact equation before he knew about Weber's findings (Fechner, 1851). Thus Fechner's rebuttal to Elsas reveals that, in contrast to what most of Fechner's critics had assumed, the passage from Weber's law (ii) to the differential equation (iii) via the infamous 'mathematical auxiliary principle' was not the spine of Fechner's derivation of the logarithmic formula.

In his reply, Elsas (1888a) had to admit that he could not find eq. (ii) in any of Fechner's writings (Elsas, 1888a, p. 132, fn. 1). However, he was not particularly impressed. Even if I cannot do full justice of Elsas' thirty-page defense here, I suspect that he did not fully realize the importance of Fechner's counter-move. Elsas went on to criticize Fechner's derivation starting from a more general finite formula, again attacking the 'mathematical auxiliary principle', that, he claimed, Fechner used more as the wand of a magician, rather than the tool of a craftsman (Elsas, 1888a, p. 134). Elsas showed that one can actually obtain the logarithmic equation (vii) without magically deriving it from the finite formula via the auxiliary principle. In this setting, the proportionality factor k becomes something that has to be obtained by differentiation of the logarithmic equation (v) and not introduced from the outset in Weber's eq. (ii) (Elsas, 1888a, p. 136). Thus, the proportionality between j.n. difference sensations and sensation differences is strictly valid only "when the change of [γ] and [β] are infinitesimally small" (Elsas, 1888a, p. 137). This, however, might have been the exact point Fechner wanted to make. The constant k is derived from the logarithmic law and not taken from the empirical Weber law. It is the logarithmic law itself, taken as measurement formula, that *defines* the relation between two units of measure (Heidelberger, [1993] 2004, p. 206). Elsas did not seem to realize that this might have fended off many neo-Kantian objections.

However, for Elsas, the essential philosophical point has not changed. Let's concede to Fechner that the number of 'sensation differences' becomes approximately proportional to the number of 'difference sensations', the smaller the stimuli difference becomes. Still, Fechner uncritically assumed that one can measure sensation by adding up sensation differences. However, it is far from obvious that sensation has such an additive structure (Elsas, 1888a, p. 139ff.). Musical intervals are certainly quantitatively comparable (a 4th is smaller than a 5th), but cannot be meaningfully added (a 5th plus a 4th equals an octave) (Elsas, 1888a, p. 140). Fechner, Elsas points out, wants us to believe that sensation is "something spiritual, psychical, and nevertheless has magnitude that can be connected additively with other magnitudes of the same types"; but Elsas simply could not understand this: "[f]undamental epistemological views prevent me from doing so" (Elsas, 1888a, p. 143). The epistemological difference between Fechner and Elsas, I think, boils down to the following: the transformation of a class of intensities into a class of measurable intensities was for Fechner a matter of definition, whereas for Elsas it was a matter of physics. Thus, for Fechner the 'difference hypothesis' was a better definition than the alternative, whereas Elsas spent the rest of the paper trying to prove that the 'ratio hypothesis' was physically superior.

When Elsas' rebuttal appeared in the 1888 issue of the *Philosophische Monatshefte*, Fechner had already passed away. "The previous remarks," Elsas acknowledged in a short postscript to his paper, "are no laurel wreath and no palm branches worth being placed on Fechner's fresh grave" (Elsas, 1888a, p. 155). They were written as Fechner was still alive and still speak as if he were alive, "since Fechner's contribution to science will never die" (Elsas, 1888a, p. 155). Elsas' note was not simply rhetorical. In the same year, Elsas dedicated a long homage to the great scientist in the national-liberal magazine *Die Grenzboten* (Elsas, 1888b). Granting Fechner the 'honors of war', Elsas now celebrated Fechner's work as the first serious critical reflection on the measurability of mental contents (cf. Elsas, 1888a, p. 114). However, Elsas' conciliatory remarks could not and probably were not meant to bridge the wide philosophical divide that separated him from Fechner. For Fechner, 'measuring sensations' was a worthy enterprise, because, as Heidelberger has pointed out, he ultimately conceived of physical measurement itself as nothing but the refinement of the resolution power of sensations via measuring apparatuses (Heidelberger, [1993] 2004, p. sec. 6.5 and in particular p. 246f.); but this point of view was fundamentally foreign to Elsas, who, like most 19th century physicists, regarded sensations as nothing but anthropomorphic slags which physical measurement was meant to get rid of.

5. Conclusion: From the Cohen Circle to the Marburg School

In those years, the Marburg critique of scientific psychology was taking a more philosophically sophisticated form in Natorp's hands (Natorp, 1887; Natorp, 1888; Natorp, 1893; cf. Luft, 2009); Elsas' technical objections against psychophysics were largely forgotten. They are mentioned, albeit rarely, in the historical literature on psychophysics and measurement theory (Heidelberger, [1993] 2004; Darrigol, 2003), though, surprisingly, never in the literature on Marburg Neo-Kantianism. Yet Elsas' work on the topic was respected in the scientific community. Beside Fechner's own detailed reply, Elsas' booklet deserved the mention (alongside Paul du Bois-Reymond's *Allgemeine Functionentheorie* (Bois-Reymond, 1882)) of his teacher Helmholtz in his 1887 paper, *Zählen und Messen* (Helmholtz, 1887). The reference to Elsas reveals that Helmholtz's paper—which is now regarded as a classic contribution to the philosophical/mathematical reflection on the notion of 'measurement' in science (Darrigol, 2003)—was probably occasioned by the question of the measurability of sensation (Heidelberger, 1993). Thus, the publications of the early 'Cohen circle' were perceived as contributions to this debate. By contrast, Cohen's commentaries on Helmholtz's 1887 paper (Cohen, 1888)—which appeared in the same 1888 volume of the *Philosophische Monatshefte* as Elsas' rebuttal to Fechner—give the impression that he did not fully recognize the philosophical importance of the issue at stake. Elsas' 'causal' theory of measurement is not even mentioned; most of all, Cohen, intent on attacking Helmholtz's naive empiricism, does not seem to appreciate the implications of the latter's analysis of the conditions governing extensive magnitudes; similarly he quickly passed over Helmholtz's definition of intensive magnitude as coefficients (measurable only as ratios between extensive magnitudes) (Helmholtz, 1887, p. 47). Instead, Cohen concluded the paper by 'plugging' his own work on the intensive magnitude as infinitesimals (Cohen, 1888, p. 273), which seems rather out of context.

In the following years, Elsas, a close friend of Heinrich Hertz (Fölsing, 1997, p. 423f.), published extensively on the new Maxwell-Hertz theory; he introduced an autonomous circuit breaker, alternative to the 'Wagner hammer' (Elsas, 1889b), suggested methods to measure electric resistance (Elsas, 1891c) and the dielectric constants (Elsas, 1891b), etc. However—despite Hertz's advice not to go astray "in swampy borderlands" like psychophysics (Hertz to Elsas, Feb. 10, 1889; cit. in Fölsing, 1997, p. 423)—he did not forgo some interesting philosophical escapades (Elsas, 1889a). After Hertz failed to find a position for him in Bonn (Fölsing, 1997, p. 429f.), he became an extraordinary professor without salary in Marburg. In a difficult financial situation (Lasswitz to Natorp, June 4, 1895; cit. in Holzhey, 1986a, p. 24, fn. 89), he died prematurely in 1895 of pulmonary tuberculosis.

Cohen's funeral oration (Cohen, 1895) in his honor is also a recollection of the interdisciplinary atmosphere of the early Marburg circle. If it was Cohen's early interest in psychophysics that contributed to gathering this small group around his Marburg chair, it was Cohen's 'infinitesimal turn' which had fragmented and eventually dissolved this group. It was not just the 'traitor' Müller that did not follow Cohen's approach, but also the sincere friend Stadler—since 1892 a professor at the ETH Zurich (Beller, 2000)—; Elsas attempted to 'translate' it for non-philosophically-trained readers, but ultimately he was himself not fully convinced. It was the newcomer Lasswitz, though never an 'official' member of the Marburg group, who was the first to try to implement a watered-down version of Cohen's connection between the 'intensive' and 'infinitesimal' in his own successful work as a historian of science (Lasswitz, 1890; see Elsas, 1891aNatorp, 1891), even if his efforts were not to Cohen's complete satisfaction (Cohen, 1896, p. XLVII).

Precisely this combination of transcendental philosophy and history of science became the trademark of the Marburg community at the turn of century. At the end of 1890s—after Natorp became full professor—Cohen started to talk cautiously of a 'little school' that was forming in Marburg (Cohen to Natorp, Apr. 19, 1897; CN, p. Br. 42, 243 and Cohen to Althoff, May 8, 1897; CN, p. Br. 43, 244). The titles of the *Preisaufgaben* launched in those years testify to the historical interests that dominated this group: Aristotle and mathematics in 1894/95 (Albert Görland), Leibniz's foundation of mathematics and natural science in 1896/97 (Ernst Cassirer), Galileo's Mechanics in 1900/1901 (Enrico DePortu), Faraday's concept of matter in 1901/1903 (Otto Buek) (cf. Holzhey, 1986a, p. 1:382f.). The prize essays were transformed into often excellent dissertations and monographs (Görland, 1899; Cassirer, 1902; Portu, 1904; Buek, 1904) which represent some early examples of the Marburg-style integration of history of philosophy and history of science which soon found its most successful expression in Cassirer's *Erkenntnisproblem* (Cassirer, 1906a).

The interests in psychophysics—the attempt "to make sensation arise and increase till it comes to consciousness and becomes integral" (Cohen, 1902, p. 441)—faded in the background. However, interestingly, it did not completely disappear. The *Preisaufgabe* suggested by Natorp in 1904/1906 concerned the problem of sensation in modern psychology. The prize remained unassigned. However, in 1906 Johannes Paulsen finished a dissertation on Fechner's concept of sensation under the guidance of Cohen and Natorp (Paulsen, 1906–1907). A longer version of it was published as a booklet (Paulsen, 1907) in the first volume of the *Philosophische Arbeiten*, the series edited by Cohen and Natorp that was meant to represent (Cassirer, 1906b, p. I-III) what people began to call the 'Marburg school' (cf. Cohen, 1913–1914). Even if Paulsen's work does not add anything new to the previous neo-Kantian criticisms of psychophysics (see also Natorp, 1912a), it shows that the issue was still considered part of the legacy of the Marburg community. After all, as Cohen himself recognized in his *Nachruf* for Stadler, it was the latter's critique of Fechner's logarithmic formula that led him to take a new course in his 1883 book (Cohen, 1910a). It is probably not just by chance that a contribution of Paulsen's (1912) was included in the 1912 *Festshrift* for Cohen's seventieth birthday, testifying to the role that psychophysics played in the evolution of the latter's thought.

As we mentioned at the opening of the present paper, this was precisely what Cassirer suggested in his article (Cassirer, 1912) for the 1912 special issue of the *Kant-Studien* that was prepared for the celebration of Cohen's retirement. Cohen's interest in the problem of 'sensation' in Kant's Anticipations of Perception, Cassirer explained, led to the notion of 'stimulus' as objectified sensation; the latter in turn must be ultimately thought of as a 'motion'. It was the question of the possibility of 'motion' as an object of scientific knowledge that ultimately led to Cohen's interest in the history of the 'infinitesimal method' (Cassirer, 1912, p. 260). Cohen's *Das Princip der Infinitesimal-Methode*, despite being unsuccessful as a scholarly work on the infinitesimal calculus, exemplified that peculiar amalgam of history and philosophy of science that became one of the trademarks of the 'Marburg school' (cf. Giovanelli, 2016). However, as we have tried to show, the appearance of that book also interrupted a fruitful discussion on 'measurability' in science which the 'Cohen circle' had initiated. This discussion seems to have left no trace in the major Marburg contributions to philosophy of science (Cassirer, 1910; Natorp, 1910). It was only in the 1930s, that Cassirer returned, although briefly, to the issue of measurement in physics in his "Determinismus und Indeterminismus" (Cassirer, 1936). However, he could resume the discussion from the exact point where the 19th century had left it: physical measurement, Cassirer insisted, is not simply the emancipation of sensation from the limits imposed by "the fundamental psychophysical law", "the Fechner-Weber law" (Cassirer, 1936, p. 42; tr. 1956, p. 32) by means of measuring apparatuses. The 'physical world', Cassirer claimed, is not a mere quantitative refinement of the 'sensible world'; it is a qualitatively different 'world', a ‘world of shadows’, a symbolic construction which takes the place of the fullness and color of the sensible world (Cassirer, 1936, p. 41; tr. 1956, p. 43).

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Abbreviations

A Immanuel Kant (1781). *Critik der reinen Vernunft*. 1st ed. Riga: Johann Friedrich Hartknoch, 1781. Repr. in Ak., p. 4.
Ak. Immanuel Kant (1900–). *Kant’s gesammelte Schriften*. Ed. by Preussische Akademie der Wissenschaften, Berlin-Brandenburgische Akademie der Wissenschaften, and Akademie der Wissenschaften in Göttingen. 29 vols. Berlin: Reimer, 1900–.
B Immanuel Kant (1787). *Critik der reinen Vernunft*. 2nd ed. Riga: Johann Friedrich Hartknoch, 1787. Repr. in Ak., p. 3.
CN Helmut Holzhey (1986b). *Cohen und Natorp*. Vol. 2. Basel: Schwabe, 1986.
CW Hermann Cohen (1977–). *Werke*. Ed. by Helmut Holzhey. 15 vols. Hildesheim: Olms, 1977–.
ECW Ernst Cassirer (1998–). *Gesammelte Werke. Hamburger Ausgabe*. Ed. by Birgit Recki. 26 vols. Hamburg: Meiner, 1998–.

References

Beiser, Frederick C. (2014). *The genesis of Neo-Kantianism, 1796–1880*. Oxford: Oxford University Press, 2014.
Beller, Mara (2000). “Kant’s impact on Einstein’s thought”. In: *Einstein. The formative years 1879–1909*. Ed. by Don Howard and John Stachel. Boston: Birkhäuser, 2000, 83–106.
Bois-Reymond, Paul Du (1882). *Allgemeine Functionentheorie*. Tübingen: Laupp, 1882.
Brentano, Franz (1874). *Psychologie Vom Empirischen Standpunkt*. Leipzig: Duncker & Humblot, 1874.
Buek, Otto (1904). “Die Atomistik und Faradays Begriff der Materie”. *Archiv für Geschichte der Philosophie* 18, 65–139.
Cantor, Georg (1884). “Review of Cohen, *Das Princip der Infinitesimal-Methode* (Cohen, 1883)”. *Deutsche Literaturzeitung* 5, 266–8.
Cassirer, Ernst (1902). *Leibniz’ System in seinen wissenschaftlichen Grundlagen*. Marburg: Elwert, 1902. Repr. in ECW, p. Vol. 1.
— (1906a). *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit*. Vol. 1. Berlin: B. Cassirer, 1906. Repr. in ECW, p. Vol. 2.
— (1906b). *Der kritische Idealismus und die Philosophie des ‘gesunden Menschenverstandes’*. Giessen: Töpelmann, 1906.
— (1910). *Substanzbegriff und Funktionsbegriff. Untersuchungen über die Grundfragen der Erkenntniskritik*. Berlin: Bruno Cassirer, 1910. Repr. in ECW, p. Vol. 6.
— (1912). “Hermann Cohen und die Erneuerung der Kantischen Philosophie”. *Kant-Studien* 17, 252–273. Repr. in ECW, p. Vol. 9, 119–138.
— (1936). “Determinismus und Indeterminismus in der modernen Physik. Historische und systematische Studien zum Kausalproblem”. *Göteborgs Högskolas Årsskrift* 42 3. Repr. in ECW, p. Vol. 19.
— (1956). *Determinism and indeterminism in modern physics. Historical and systematic studies of the problem of causality*. New Haven: Yale University Press, 1956.
Cohen, Hermann (1868). “Mythologische Vorstellungen von Gott und Seele”. *Zeitschrift Völkerpsychologie und Sprachwissenschaft* 5, 396–434. Repr. in CW, p. Vol. 12, 271–343.
— (1871). *Kants Theorie der Erfahrung*. 1st ed. Berlin: Dümmler, 1871. Repr. in CW, p. , vol. 1/III.
— (1877). *Kants Begründung der Ethik*. 1st ed. Berlin: Dümmler, 1877. The 2nd edition (Cohen, 1910b) is reprinted in CW, vol. 2.

Cohen, Hermann (1883). *Das Princip der Infinitesimal-Methode und seine Geschichte. Ein Kapitel zur Grundlegung der Erkenntnisskritik*. Berlin: Dümmler, 1883. Repr. in in CW, vol. 5/I.

— (1885). *Kants Theorie der Erfahrung*. 2nd ed. Berlin: Dümmler, 1885. Repr. in in CW, vol. 1/I.

— (1888). "Jubiläums-Betrachtungen". *Philosophische Monatshefte* 24, 257–291. Repr. in Cohen, 1928, p. 1:397–431.

— (1895). "Worte an der Bahre von Adolf Elsas, Professor der Physik an der Universität Marburg". Gesprochen im Trauerhause am 16. Mai 1895. In: *Schriften zur Philosophie und Zeitgeschichte*. Ed. by Albert Gorland and Ernst Cassirer. Berlin: B. Cassirer, 1895, 396–397.

— (1896). "Einleitung mit kritischem Nachtrag". In: Lange, Friedrich Albert. *Geschichte des Materialismus und Kritik seiner Bedeutung in der Gegenwart*. 5th ed. Leipzig: Baedeker, 1896. Repr. in CW, p. Vol. 5/2.

— (1902). *Logik der reinen Erkenntnis*. Berlin: B. Cassirer, 1902. Repr. in CW, p. vol. 6.

— (1910a). "August Stadler. Ein Nachruf". *Kant-Studien* 15, 403–420. Repr. in CW, p. vol. 15, 513–542.

— (1910b). *Kants Begründung der Ethik. Nebst ihren Anwendungen auf Recht, Religion und Geschichte*. 2nd ed. Berlin: B. Cassirer, 1910.

— (1913–1914). "Paul Natorp. Zu seinem 60. Geburtstag". In: *Schriften zur Philosophie und Zeitgeschichte*. Ed. by Albert Gorland and Ernst Cassirer. Berlin: B. Cassirer, 1913–1914, 466–470.

— eds. (1928). *Schriften zur Philosophie und Zeitgeschichte*. Berlin: B. Cassirer, 1928.

— (2015). *Briefe an August Stadler*. Ed. by Hartwig Widebach. Basel: Schwabe, 2015.

Darrigol, Olivier (2003). "Number and measure. Hermann von Helmholtz at the crossroads of mathematics, physics, and psychology". *Studies in the history of the philosophy of science* 34, 515–73.

Delbœuf, Joseph (1873). *Études psychophysiques sur la mesure des sensations*. Bruxelles: Muquardt, 1873.

— (1878). "La loi psychophysique et le nouveau livre de Fechner". *Revue philosophique de la France et de l'etranger* 5, 34–63.

Delezenne, Charles Edouard Joseph (1826). "Mémoire sur les valeurs numériques des notes de la gamme". *Recueil des travaux de la Societè des Sciences de l'Agriculture et des Arts de Lille 1826 et premier semestre 1827* 4, 1–56.

Dzhafarov, Ehtibar N. and Hans Colonius (1999). "Fechnerian metrics in unidimensional and multidimensional stimulus spaces". *Psychonomic bulletin & review* 6 2, 239–68.

Eccarius, Wolfgang (1985). "Georg Cantor und Kurd Laßwitz. Briefe zur Philosophie des Unendlichen". *NTM – Schriftenreihe für Geschichte der Naturwissenschaften, Technik und Medizin* 22 1, 29–52.

Elsas, Adolf (1881). *Ueber erzwungene Schwingungen weicher Fäden*. Würzburg: Elberfeld/Fassbender, 1881.

— (1882). *Untersuchungen über erzwungene Membranschwingungen*. Marburg: Blochmann & Sohn, 1882.

— (1883a). "Review of F. A. Müller, *Das Axiom der Psychophysik* (F. A. Müller, 1882)". *Zeitschrift für Philosophie und Philosophische Kritik* 83, 126–134.

— (1883b). "Untersuchungen über erzwungene Schwingungen von Platten". *Annalen der Physik und Chemie*. new 20 11, 468–484.

— (1884). "Review of Cohen, *Das Princip der Infinitesimal-Methode* (Cohen, 1883)". *Philosophische Monatshefte* 20, 556–560.

— (1885a). "Bemerkungen zu der Abhandlung des Herrn F. Melde. Akustische Experimentaluntersuchungen". *Annalen der Physik und Chemie*. new 25 8, 676–677.

— (1885b). "Review of Stadler, *Kants Theorie der Materie* (Stadler, 1883)". *Philosophische Monatshefte* 21, 144–160.

— (1886a). *Der Schall. Eine populäre Darstellung der physikalischen Akustik mit besonderer Berücksichtigung der Musik*. Leipzig/Prag: Freytag/Tempsky, 1886.

— (1886b). *Über die Psychophysik. Physikalische und erkenntnisstheoretische Betrachtungen*. Marburg: Elwert, 1886.

— (1886c). "Ueber die Nobili'schen Farbenringe und verwandte electrochemische Erscheinungen". *Annalen der Physik und Chemie*. new 29 10, 331–343.

— (1887). "Ueber die Nobili'schen Farbenringe und verwandte electrochemische Erscheinungen". *Annalen der Physik und Chemie*. new 30 4, 620–630.

— (1888a). "Die Deutung der psychophysischen Gesetze". *Philosophische Monatshefte* 24, 129–155.

— (1888b). "Zum Andenken Gustav Theodor Fechners". *Die Grenzboten. Zeitschrift für Politik, Literatur und Kunst* 47 2, 73–80, 113–124.

— (1889a). "Kritische Betrachtungen über die Wahrscheinlichkeitsrechnung". *Philosophische Monatshefte* 25, 557–584.

— (1889b). "Ueber einen selbstthätigen Stromunterbrecher". *Annalen der Physik und Chemie*. new 37 8, 675–680.

— (1891a). "Review of Lasswitz, *Geschichte der Atomistik* (Lasswitz, 1890)". *Zeitschrift für Philosophie und Philosophische Kritik* 99, 290–303.

— (1891b). "Ueber eine neue Methode zur Bestimmung von Dielectricitätsconstanten". *Annalen der Physik und Chemie*. new 280 12, 654–665.

Elsas, Adolf (1891c). "Ueber Widerstandsmessungen mit Hülfe des Telephons". *Annalen der Physik und Chemie* 44 12, 666–680.

Fechner, Gustav Theodor (1851). *Zend-Avesta oder über die Dinge des Himmels und des Jenseits. Vom Standpunkt der Naturbetrachtung.* 3 vols. Hamburg: Voss, 1851.

— (1860). *Elemente der Psychophysik.* 2 vols. Leipzig: Breitkopf & Härtel, 1860. first volume translated in Fechner, 1966.

— (1877). *In Sachen der Psychophysik.* Leipzig: Breitkopf und Härtel, 1877.

— (1882). *Revision der Hauptpuncte der Psychophysik.* Leipzig: Breitkopf & Härtel, 1882.

— (1888). "Ueber die psychischen Maßprincipien und das Weber'sche Gesetz. Discussion mit Elsas und Köhler". *Philosophische Studien* 4. Ed. by Wilhelm Wundt, 161–230. Partially translated in Fechner, 1987.

— (1966). *Elements of psychophysics.* Ed. by Davis H. Howes and Edwin G. Boring. Trans. by Helmut E. Adler. With an intro. by Edwin G. Boring. Vol. 1. New York: Holt, Rinehart/Winston, 1966.

— (1987). "My own viewpoint on mental measurement (1887)". Trans. by Eckart Scheerer. *Psychological research* 49 4, 213–219.

Fölsing, Albrecht (1997). *Heinrich Hertz. Eine Biographie.* Hamburg: Hoffmann und Campe, 1997.

Giovanelli, Marco (2016). "Hermann Cohen's *Das Princip der Infinitesimal-Methode*. The History of an Unsuccessful Book". *Studies in History and Philosophy of Science Part A* 58, 9–23.

Görland, Albert (1899). *Aristoteles und die Mathematik.* Marburg: Elwert, 1899.

Heidelberger, Michael (1993). "Fechner's impact for measurement theory". *Behavioral and brain sciences* 16 1, 146–148.

— (2004). *Nature from within. Gustav Theodor Fechner and his psychophysical worldview.* Pittsburgh: University of Pittsburgh Press, 2004 (1993). Trans. of *Die innere Seite der Natur. Gustav Theodor Fechners wissenschaftlich-philosophische Weltauffassung.* Frankfurt am Main: Klostermann, 1993.

— (2010). "Functional relations and causality in Fechner and Mach". *Philosophical psychology* 23 2, 163–172.

Helmholtz, Hermann von (1867). *Handbuch der physiologischen Optik.* Leipzig: Voss, 1867.

— (1877). *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik.* Braunschweig: Vieweg, 1877.

— (1887). *Zählen und Messen, erkenntnisstheoretisch betrachtet.* In: *Philosophische Aufsätze, Eduard Zeller zu seinem fünfzigjährigen Doctorjubiläum gewidmet.* Leipzig: Fues, 1887, 17–52.

Hering, Ewald (1875). *Zur Lehre von der Beziehung zwischen Leib und Seele. I. Mittheilung: Über Fechner's psychophysisches Gesetz.* In: *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften in Wien* vol. 72 (III Abt.) 310–348.

Holzhey, Helmut (1986a). *Cohen und Natorp.* 2 vols. Basel: Schwabe, 1986.

Kant, Immanuel (1786). *Metaphysische Anfangsgründe der Naturwissenschaft.* Riga: Johann Friedrich Hartknoch, 1786. Repr. in in Ak. 4:465-565.

Köhler, Alfred (1886). "Ueber die hauptsächlichen Versuche einer mathematischen Formulirung des psychophysischen Gesetzes von Weber". *Philosophische Studien* 3, 572–642.

Kries, Johannes von (1882). "Ueber die Messung intensiver Grössen und über das sogenannte psychophysische Gesetz". *Vierteljahrsschrift für wissenschaftliche Philosophie* 4 3, 257–294.

Lange, Friedrich Albert (1866). *Geschichte des Materialismus und Kritik seiner Bedeutung in der Gegenwart.* 1st ed. Iserlohn: Baedeker, 1866.

— (1968). *Über Politik und Philosophie. Briefe und Leitartikel 1862 bis 1875.* Ed. by Georg Eckert. Duisburg: Braun, 1968.

Lasswitz, Kurd (1878). *Atomistik und Kriticismus. Ein Beitrag zur erkenntnistheoretischen Grundlegung der Physik.* Vieweg: Braunschweig, 1878.

— (1884). "Giordano Bruno und die Atomistik". *Vierteljahrsschrift für wissenschaftliche Philosophie* 8, 18–55.

— (1885a). "Review of Cohen, *Das Princip der Infinitesimal-Methode* (Cohen, 1883)". *Vierteljahrsschrift für wissenschaftliche Philosophie und Soziologie* 9, 494–503.

— (1885b). "Zur Rechtfertigung der kinetischen Atomistik". *Vierteljahrsschrift für wissenschaftliche Philosophie* 9, 137–161.

— (1887a). "Review of Elsas, *Über die Psychophysik* (Elsas, 1886b)". *Deutsche Literaturzeitung* 8, 3–4.

— (1887b). "Review of F. A. Müller, *Das Problem der Continuität im Mathematik und Mechanik* (F. A. Müller, 1886)". *Deutsche Literaturzeitung* 28, 1003–1004.

— (1888). "Das Problem der Continuität". *Philosophische Monatshefte* 24, 9–36.

— (1890). *Geschichte der Atomistik.* 2 vols. Hamburg/Leipzig: Voss, 1890.

Luft, Sebastian (2009). "Reconstruction and reduction. Natorp and Husserl on method and the question of subjectivity". In: *Neo-kantianism in contemporary philosophy.* Ed. by Rudolf A. Makkreel and Sebastian Luft. Bloomington: Indiana University Press, 2009, 59–91.

Luft, Sebastian, ed. (2015). *The neo-Kantian reader*. London/New York: Routledge, 2015.

Mach, Ernst (1863). *Vorträge über Psychophysik*. Wien: Sommer, 1863.

Makkreel, Rudolf A. and Sebastian Luft, eds. (2009). *Neo-Kantianism in contemporary philosophy*. Bloomington: Indiana University Press, 2009.

Michell, Joel (1999). *Measurement in psychology. Critical history of a methodological concept*. Cambridge: Cambridge Univerity Press, 1999.

Müller, Ferdinand August (1882). *Das Axiom der Psychophysik und die psychologische Bedeutung der Weber'schen Versuche*. Marburg: Elwert, 1882.

— (1886). *Das Problem der Continuität im Mathematik und Mechanik. Historische und systematische Beiträge*. Elwert: Marburg, 1886.

Müller, Georg Elias (1878). *Zur Grundlegung der Psychophysik. Kritische Beiträge*. Berlin: Grieben, 1878.

Natorp, Paul (1881). "Leibniz und der Materialismus". 1881. First pub. in Helmut Holzhey. "Paul Natorp. Leibniz und der Materialismus (1881)". *Studia Leibnitiana* 17 1, 3–14.

— (1882a). *Descartes' Erkenntnisstheorie. Eine Studie zur Vorgeschichte des Kriticismus*. Marburg: Elwert, 1882.

— (1882b). "Die kosmologische Reform des Kopernikus in ihrer Bedeutung für die Philosophie". *Preussische Jahrbücher* 71, 355–375.

— (1882c). "Galilei als Philosoph. Eine Skizze". *Philosophische Monatshefte* 18, 193–229.

— (1887). "Über objective und subjective Begründung der Erkenntnis". *Philosophische Monatshefte* 23 23, 257–286.

— (1888). *Einleitung in die Psychologie nach kritischer Methode*. Freiburg: Mohr, 1888.

— (1891). "Review of Lasswitz, *Geschichte der Atomistik* (Lasswitz, 1890)". *Philosophische Monatshefte* 27, 334.

— (1893). "Zu den Vorfragen der Psychologie". *Philosophische Monatshefte* 29, 581–611.

— (1910). *Die logischen Grundlagen der exakten Wissenschaften*. Leipzig Berlin: Teubner, 1910.

— (1912a). *Allgemeine Psychologie nach kritischer Methode*. Tübingen: Mohr, 1912.

— (1912b). "Kant und die Marburger Schule". *Kant-Studien* 17, 193–221.

Paulsen, Johannes (1906–1907). "Der Begriff der Empfindung bei Fechner". Inauguraldissertation. Universität Marburg, 1906–1907.

— (1907). *Das Problem der Empfindung*. Töpelmann: Giessen, 1907.

— (1912). "Ueber das empirische Verhältnis von Reiz und Empfindung". In: *Philosophische Abhandlungen. Hermann Cohen zum 70. Geburtstag (4. Juli 1912) dargebracht*. Berlin: B. Cassirer, 1912, 123–135.

Philippi, E. (1883). "Review of F. A. Müller, *Das Axiom der Psychophysik* (F. A. Müller, 1882)". *Philosophische Monatshefte* 19, 574–587.

Plateau, Joseph (1872). "Sur la mesure des sensations physiques et sur la loi qui lie l'intensité de ces sensations à l'intensité de la cause excitante". *Bulletins de l'Academie Royale des Sciences, des Lettres et des Beaux-Arts de Belgique. 2me Serie* 34, 141–142.

Poma, Andrea (1997). *The critical philosophy of Hermann Cohen*. Albany, NY: State University of New York Press, 1997 (1989). Trans. of *La filosofia critica di Hermann Cohen*. Milano: Mursia, 1989.

Portu, Enrico Stefano Maria de (1904). *Galileis Begriff der Wissenschaft*. Marburg: Elwert, 1904.

Scheerer, Eckart (1987). "The unknown Fechner". *Psychological research* 49 4, 197–202.

Sieg, Ulrich (1994). *Aufstieg und Niedergang des Marburger Neukantianismus. Die Geschichte einer philosophischen Schulgemeinschaft*. Würzburg: Königshausen und Neumann, 1994.

Stadler, August (1874). *Kants Teleologie und ihre erkenntnisstheoretische Bedeutung. Eine Untersuchung*. Berlin: Dümmler, 1874.

— (1876). *Die Grundsätze der reinen Erkenntnisstheorie in der Kantischen Philosophie. Kritische Darstellung*. Leipzig: Hirzel, 1876.

— (1878). "Über die Ableitung des Psychophysischen Gesetzes". *Philosophische Monatshefte* 14, 215–23.

— (1880). "Das Gesetz der Stetigkeit bei Kant". *Philosophische Monatshefte* 16, 577–97.

— (1883). *Kants Theorie der Materie*. Leipzig: Hirzel, 1883.

Stevens, Stanley Smith (1961). "To honor Fechner and repeal his law". *Science* 133 3446, 80–86.

Tannery, Jules (1875a). "Correspondence. A propos du logarithme des sensations". *La Revue scientifique* 15, 876–877.

— (1875b). "La mesure des sensations. Réponses à propos du logarithme des sensations". *La Revue scientifique* 8, 1018–1020.

Weber, Ernst Heinrich (1834). *De pulsu, resorptione, auditu et tactu. Annotationes anatomicae et physiologicae*. Leipzig: Koehler, 1834.

— (1846). *Tastsinn und Gemeingefühl*. In: *Handwörterbuch der Physiologie mit Rücksicht auf physiologische Pathologie*. Ed. by Rudolph Wagner. Vol. 3.2. Braunschweig: Vieweg, 1846, 481–588.

Wundt, Wilhelm (1874). *Grundzüge der physiologischen Psychologie*. Leipzig: Engelmann, 1874.

Zeller, Eduard (1881). "Über die Messung psychischer Vorgänge". *Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin. Philosophisch-Historische Klasse* 3, 1–16.

Zeller, Eduard (1882). *Einige weitere Bemerkungen über die Messung psychischer Vorgänge*. In: *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* vol. 1, 295–305.