Confirmation via Analogue Simulation: A Bayesian Analysis

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Abstract

Analogue simulation is a novel mode of scientific inference found increasingly within modern physics, and yet all but neglected in the philosophical literature. Experiments conducted upon a table-top 'source system' are taken to provide insight into features of an inaccessible 'target system', based upon a syntactic isomorphism between the relevant modelling frameworks. An important example is the use of acoustic 'dumb hole' systems to simulate gravitational black holes. In a recent paper it was argued that there exists circumstances in which confirmation via analogue simulation can obtain; in particular when the robustness of the isomorphism is established via universality arguments. The current paper supports and extends these claims via an analysis in terms of Bayesian confirmation theory.

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1 Introduction

The concept of 'Analogue Simulation', introduced in (Dardashti et al. 2015), can be understood as a particular mode of analogical reasoning: it involves making inferences about one physical system by consideration of another physical system that shares certain common features. In a case of analogue simulation the analogical relationship between the two systems is established via a 'syntactic isomorphism' between the modelling frameworks used to describe each system. Such relationships can be found, for example, between models of black holes and fluid mechanical analogue 'dumb holes' (Unruh 1981) and certain applications of relativistic quantum mechanics and models of trapped ions (Gerritsma et al. 2010). Significantly, within these pairs of systems, one half is typically either experimentally inaccessible (e.g. black holes) or displays phenomena that are difficult or impossible to test (e.g. *Zitterbewegung* in relativistic quantum mechanics). Thus, it is, prima facie, understandable why experimental scientists have been pursuing the practice of analogue simulation with such enthusiasm (Steinhauer 2014).

Analogue simulation is a new inferential tool found at the cutting edge of modern science that we see good reasons to take as potentially transformative. In this paper we will use a Bayesian analysis to explicate the formal structure of inferences that analogue simulation can allow us to make. It was argued in (Dardashti et al. 2015) that there is a difference of kind between the type of inferences that can be licensed by 'Analogue Simulation' and those that are supported by more traditional 'Arguments by Analogy'. In particular, unlike in the case of arguments by analogy, it was claimed that there exists circumstances in which 'confirmation via analogue simulation' can obtain. The two key results of this paper are: i) support for the confirmation claim via the proof of two relevant theorems; and ii) a formal model for 'multiple source' analogue simulation displaying the generic feature of 'saturation' in confirmatory power.

2 Analogue Simulation

The particular importance of analogue simulation derives from the relative strength of inferences that it licences. We can compare a case of analogue simulation with a prototypical analogical argument, where two systems share some positively analogical feature, and this similarity is used to infer the existence of further similarities.[1] Typically analogical arguments have the form of abstract speculative inferences regarding possible features of one system ('the target') based on known features of another system ('the source'). Classic examples are Reid's argument for the existence of life on other planets based upon life on earth (Reid and Hamilton 1850) or Hume's argument for animal consciousness based upon human consciousness (Hume 1978).

Analogical arguments can play an important heuristic role in scientific practice: they can provide 'cognitive strategies for creative discovery' (Bailer-Jones 2009, p.56). Analogical arguments are not, however, typically taken to be able to provide Bayesian confirmation of a hypothesis regarding the target system. From a Bayesian perspective on confirmation, evidence for a hypothesis can count as confirmatory only if the probability of the hypothesis given the evidence together with certain background assumptions is larger than the probability of the hypothesis given only the background assumptions. In a persuasive analysis, Bartha (2010, 2013) contends that we should take the information encapsulated in an analogical argument to already be part of the background knowledge, and thus the probability of a hypothesis regarding the target system to be identical before and after including the information encapsulated in an analogical argument. Rather one assumes that arguments by analogy to establish only the *plausibility* of a conclusion, and with it grounds for further investigation (Salmon 1990; Bartha 2010). On this analysis, it is not *in principle* possible for analogical arguments to confer inductive support for a hypothesis. That is, although analogical arguments can certainly be stronger or weaker, even the strongest possible analogical argument cannot confer confirmation in a Bayesian sense: they are abstract inferences that can only ever support plausibility claims rather than providing inductive evidence.

Analogue simulation, by contrast, is an essentially empirical activity: it is a technique for learning about the world by manipulating it. In order for the isomorphisms between modelling frameworks to be constructed it is typically the

[1] The literature on analogical reasoning in science is fairly extensive, with particularly noteworthy contributions by Keynes (1921), Hesse (1964, 1966), (2009) and Bartha (2010, 2013). See also Norton (2011) for an importantly different take on analogical arguments. Norton's analysis focuses on analogical arguments that proceed via subsumption of the target system into a larger class of entities, including the source system. There are interesting parallels between the structure of such inferences and our notion of analogue simulation supported by universality arguments.

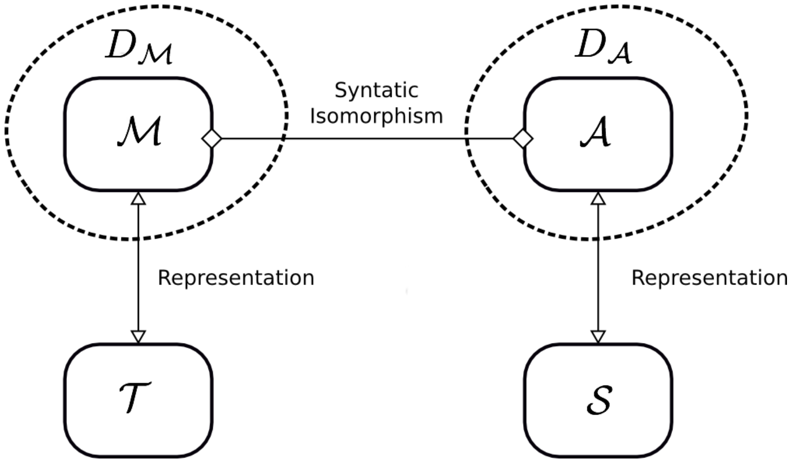

Figure 1: Analogue simulation schema.

case that the source system has to be specifically prepared so that the relevant equations match those that describe the target system. Essentially, we manipulate the source such that certain explicit modelling assumptions, matching those for the target, obtain. Analogue simulation thus resembles a form of experimentation, involving the 'programming' of a physical system such that it can be used to 'simulate' another physical system. Thus, we see that conclusions from the philosophical analysis of analogical argument should not be taken to be readily extendible to cases of analogue simulation in contemporary science. Rather, we must look for a more specialised analysis to assess the philosophical foundations of analogue simulation.

Following the analysis of (Dardashti et al. 2015) we can characterise a case of analogue simulation as follows (see Figure 1). For certain purposes and to a certain degree of desired accuracy, modelling framework $\mathcal{M}$ is adequate for modelling a target system $\mathcal{T}$ within a certain domain of conditions $D_{\mathcal{M}}$. For certain purposes and to a certain degree of desired accuracy, a modelling framework $\mathcal{A}$ is adequate for modelling a source system $\mathcal{S}$ within a certain domain of conditions $D_{\mathcal{A}}$. There exists exploitable mathematical similarities between the structure of $\mathcal{M}$ and $\mathcal{A}$ sufficient to define a syntactic isomorphism within the domains $D_{\mathcal{M}}$ and $D_{\mathcal{A}}$. We are interested in knowing something about the behaviour of a system $\mathcal{T}$ within the domain of conditions $D_{\mathcal{M}}$. For whatever reasons, we are unable to directly observe the behaviour of a system $\mathcal{T}$ in those conditions to the degree of accuracy we require. We are, on the other hand, able to study a system $\mathcal{S}$, and after having put it in suitable conditions are able to detect some analogue behaviour $\mathcal{E}$.

The paradigm case of analogue simulation is the simulation of an astrophysical black hole via a fluid mechanical 'dumb hole' (Unruh 1981). For that case, $\mathcal{S}$ would be the dumb hole, $\mathcal{A}$ the relevant equations in continuum fluid mechanics, and $D_{\mathcal{A}}$ the conditions in which these equations are adequate (e.g. length scales in which the molecular nature of fluids is not important). $\mathcal{T}$ would then be an astrophysical black hole, $\mathcal{M}$ the relevant equations in semiclassical gravity, and $D_{\mathcal{M}}$ the conditions in which these equations are adequate (e.g. length scales in which the quantum nature of spacetime is not important). In this case the system $\mathcal{T}$ is inaccessible, since we are unable to probe astrophysical black holes experimentally. A particular phenomenon we would like to test is Hawking's famous prediction that black holes radiate (Hawking 1975). The key virtue of the analogue models of black holes is then that there is a fluid analogue of Hawking radiation, $\mathcal{E}$. Recent experiments (Steinhauer 2014) point towards observation of the relevant $\mathcal{E}$. Can the framework of analogue simulation be used to justify the claim that we have 'confirmed' the phenomenon in the target system?

In both this specific case and in general there are reasons to be sceptical. This is because, as noted in (Dardashti et al. 2015), the fact that $\mathcal{A}$ is adequate for $\mathcal{S}$ in $D_{\mathcal{A}}$ need not be connected to the question of whether $\mathcal{M}$ is adequate for $\mathcal{T}$ in $D_{\mathcal{M}}$. Prima facie, we have no good reason, for example, to expect that the length scale at which a fluid model breaks down because of the molecular nature of fluids is connected to the length scale at which a black hole model breaks down because of the quantum nature of spacetime. The prima facie independence of the two domains of conditions means that the behaviour of the analogue system might not be a good guide to the behaviour of the target system, even if the relevant syntactic isomorphism holds. In such circumstances it would not be appropriate to claim one can have confirmation via analogue simulation: we have no reason to believe that an analogue behaviour $\mathcal{E}$ is relevant evidence for the behaviour of the system $\mathcal{T}$. What we need are additional arguments that connect the realisation of the two domains of conditions, and thus imply that the relevant syntactic isomorphism holds between adequate modelling frameworks. It is only then that we can argue that the analogue behaviour $\mathcal{E}$ is relevant evidence for the behaviour of the system $\mathcal{T}$, and thus that there can be confirmation via analogue simulation.

Let us consider the nature of possible arguments that would connect the realisation of the domains of conditions, $D_{\mathcal{M}}$ and $D_{\mathcal{A}}$. The most plausible candidates for such arguments would come from additional knowledge of the underlying physics that are supported by empirical evidence. The arguments would thus be 'model-external', in the sense that they are arguments given in addition to the modelling frameworks $\mathcal{M}$ and $\mathcal{A}$, and they would also be 'empirically grounded'. We will abbreviate such arguments 'MEEGA'. An important example of such MEEGA, relevant to the 'dumb hole' simulation of Hawking radiation, are universality arguments. Such arguments are based upon the insensitivity of the Hawking flux to arbitrary modifications of the dispersion relations used in both the dumb hole and black hole models (Unruh and Schützhold 2005). In (Dardashti et al. 2015) it was argued that these universality arguments can be used to ground claims for the confirmation of gravitational Hawking radiation given its simulation in analogue 'dumb hole' experiments. Other potential examples of MEEGA are: i) arguments from one theory, such as Quantum Field Theory in the context of the Newton-Coulomb syntactic isomorphism , see §3.2 of (Dardashti et al. 2015)); ii) arguments from two theories, such as the relationship between electric current and fluid flow in certain special circumstances (Logan 1962); and iii) arguments based on piecemeal adjustment of the computational scheme to match observed phenomena in the context of computer simulation (Winsberg 2010).

The analysis and articulation of such examples is a large project that will not be further pursued here. Rather, we would like to consider whether confirmation via analogue simulation can be established in principle, given MEEGA. To this end, in the following section, we will introduce the Bayesian framework for probabilistic confirmation and analyse a simple analogue simulation set up with a single source and some (rather abstract) notion of MEEGA, captured in a propositional variable X. We will then, in Section 4, extend this analysis to the case of analogue simulation with multiple source systems, with a view to capturing the essential inferential features of analogue simulation supported by universality arguments, such as the 'dumb hole' case.

3 Bayesian Confirmation

The key claim that we wish to investigate is that, in certain circumstances, analogue simulation can provide inductive support for a hypothesis regarding the target system, on the basis of empirical evidence regarding the source system: i.e. it can give us confirmation.[2] In what follows we give a Bayesian network representation of the proposed inferential structure of analogue simulation defended in (Dardashti et al. 2015) and show that the evidence in the source system can provide confirmation of hypotheses regarding the target system in certain circumstances.

3.1 Analogue Simulation Without Confirmation

Let us start with the representation of the target system $\mathcal{T}$. We denote by M a propositional variable that takes the two values:

M : The modelling framework $\mathcal{M}$ provides an empirically adequate description of the target system $\mathcal{T}$ within a certain domain of conditions $D_{\mathcal{M}}$.

¬M : The modelling framework $\mathcal{M}$ does not provide an empirically adequate description of the target system $\mathcal{T}$ within a certain domain of conditions $D_{\mathcal{M}}$.

The adequacy of the modelling framework $\mathcal{T}$ depends on whether the background assumptions which justify the empirical adequacy of the modelling framework obtain. We denote with X_M the random variable with the values:

[2]For models of confirmation in terms of the Bayesian framework see (Hartmann and Sprenger 2010; Bovens and Hartmann 2004) or for the hypothetic-deductive framework see (Betz 2013). Throughout this paper, we follow the convention that propositional variables are printed in italic script, and that the instantiations of these variables are printed in roman script.

X$_M$: The background assumptions $x_M = \{x_M^1, x_M^2, ..., x_M^n\}$ are satisfied for system $\mathcal{T}$.

$\neg$X$_M$: The background assumptions $x_M = \{x_M^1, x_M^2, ..., x_M^n\}$ are not satisfied for system $\mathcal{T}$.

The role of the background assumptions is to define and justify the domain of conditions for the model. These assumptions involve knowledge, both theoretical and empirical, that goes beyond what is encoded within the model. Such knowledge need not be in the form of a simple, unified framework. Rather the background knowledge of the people who build and use models can contain an incompletely integrated set of explicit and tacit ideas about when a particular modelling framework will be adequate for a particular purpose and to a particular desired degree of accuracy.

With this in mind, we can introduce the random variables A and X_A for the source system $\mathcal{S}$. Where A is a propositional variable that takes the two values:

A : The modelling framework $\mathcal{A}$ provides an empirically adequate description of the source system $\mathcal{S}$ within a certain domain of conditions $D_{\mathcal{S}}$.

$\neg$A : The modelling framework $\mathcal{A}$ does not provide an empirically adequate description of the source system $\mathcal{S}$ within a certain domain of conditions D_A.

and X_A is the random variable with the values:

X$_A$: The background assumptions $x_A = \{x_A^1, x_A^2, ..., x_A^k\}$ are satisfied for system $\mathcal{S}$.

$\neg$X$_A$: The background assumptions $x_A = \{x_A^1, x_A^2, ..., x_A^k\}$ are not satisfied for system $\mathcal{S}$.

The systems $\mathcal{T}$ and $\mathcal{S}$ are assumed to differ in terms of their material constituency and the fundamental laws governing their dynamics. This means that the background assumptions behind the models $\mathcal{M}$ and $\mathcal{A}$ can reasonably be assumed to be very different. Given this, it is justified, prima facie, to assume that X_M and X_A are probabilistically independent. Furthermore, we have assumed that the source system is empirically accessible meaning we can gain empirical evidence regarding (at least) some of its consequences. We can encode this by introducing a variable E corresponding to the two values, E, the empirical evidence obtains, and $\neg$E, the empirical evidence does not obtain.

We can represent all the variables introduced thus far as well as the probabilistic dependancies using a Bayesian network (Bovens and Hartmann 2004). The random variables are represented as 'nodes' in the network (i.e. circles) and the probabilistic dependancies as directed edges (i.e. arrows). We draw an arrow between two nodes when the variable in the 'parent node' has a direct influence on the variable in the 'child node'. Probabilistic independence

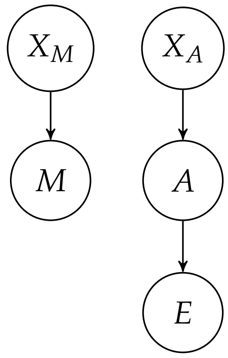

Figure 2: One source system without MEEGA.

is represented implicit by the absence of an arrow between two nodes. The entire set up thus far is represented by the Bayesian network in Figure 2.

The assumption that X_A and X_M are probabilistically independent is equivalent to us having reasons to believe that there do not exist MEEGA connecting the realisation of the domains of conditions, $D_{\mathcal{A}}$ and $D_{\mathcal{M}}$. In these circumstances, the evidence E in system $\mathcal{S}$ will be irrelevant to what we believe about the modelling framework $\mathcal{M}$. Confirmation via analogue simulation does not hold without MEEGA. This is despite the syntactic isomorphism that exists between $\mathcal{M}$ and $\mathcal{A}$. The isomorphism merely implies that there will be a term within the modelling language of $\mathcal{M}$ that is counterpart to the term within $\mathcal{A}$ that refers to $\mathcal{E}$. On our view, such a syntactic relationship between modelling frameworks has, on its own, no evidential force.

Here it might be objected that this final statement is too strong. What if, for instance, we had ten syntactically isomorphic systems, and found that nine of them operate according to the same laws. Surely this finding would raise the probability that the tenth also operates according to these laws? Does this not suggests that the possibility of these systems operating according to similar laws always had a non-zero probability?

In responding to this objection we must draw again the distinction between: i) similarity as to *syntactic form* of laws used to adequately model two systems; and ii) similarity as to the *material constituency and dynamics* of two systems. Our claim is that in circumstances where we know that the source and target system differ significantly in the second physical respect the closeness of the relationship in the first syntactic respect is not evidentially relevant. This claim stands up even to the hypothetical counter-example. Let us assume that we have ten materially and dynamically very different systems that adequately modelled by syntactically isomorphic laws. If we were to find evidence (probabilistic or otherwise) of a physical connection between nine of the ten systems of laws that would certainly encourage us to look for a physical basis for the relevant connection – we would assume, most likely, that they would be members of the same 'universality class'.[3] It would still not however, in-and-of-itself, warrant drawing a further evidential connection to the tenth system.

[3]See for example (Batterman 2000; Batterman 2002).

The tenth system is, by assumption, materially and dynamically very different to the other nine, so we cannot purely on the basis of the syntactic isomorphism assume it to be within the same 'universality class'. Membership of such classes always depends upon the physical relationship between systems, not syntactical relationships.

3.2 Analogue Simulation With Confirmation

The key idea behind analogue simulation supported by MEEGA is that there exists empirically grounded arguments that function as inferential links *between the background assumptions* of both the model of the source system and the model of the target system.

In the case where there is no MEEGA, evidence in $\mathcal{A}$ provides inductive support for the adequacy of the background assumptions X_A alone. On the other hand, with MEEGA, evidence in $\mathcal{A}$ will also support the background assumptions X_M. This support is limited to the elements of the assumptions that are actually addressed by the specific MEEGA – the 'common background assumptions'. So while MEEGA may relate two background assumptions, say x_A^1 and x_M^2, the other background assumption will, of course, remain independent unless they are linked via another MEEGA. The inductive support that analogue simulation with MEEGA will then provide for M will depend on how certain we are about the adequacy of the related background assumptions. If we have already independent grounds on which to assign high probabilities to x_A^1 or x_M^2, then there is not much added in terms of inductive support we gain through MEEGA. In the context of simulating Hawking radiation via 'dumb holes', the universality argument provides the link between the background assumptions, which is here the independence of the phenomenon in each system from the respective influences of the higher energy theories. The independence claim, however, is probably the least supported of the background assumption and is thus the route via which analogue simulation can provide strong inductive support for black hole Hawking radiation.[4]

In order to make explicit calculation tractable we will subsume both the MEEGA and the common background assumptions within a single variable X. The binary value X has the values:

X : there exist MEEGA in support of common background assumptions.

¬X : there does not exist MEEGA in support of common background assumptions

So X expresses a rather general claim, which can plausibly be assumed to be uncertain. If we were certain about X, the inference from $\mathcal{A}$ to $\mathcal{M}$ would be blocked. We will say more about this later. We will also subsume the remaining background assumptions, that is those that are not addressed by MEEGA, under the nodes M and A.

[4]Unruh for instance even asks that 'if the derivation relies on such absurd physical assumptions, can the result be trusted?' (Unruh 2014, p.534)

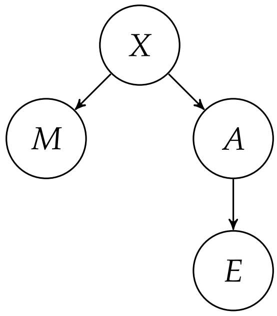

Figure 3: Simplified network for one source system with MEEGA.

Under the conditions of our assumptions, the simplified Bayesian network given in Figure 3 will then adequately model the chain of inferences involved in analogue simulation supported by MEEGA. We would like to show that E confirms M within a Bayesian theory of confirmation. This requires that one proves that $P(\mathrm{M}|\mathrm{E}) > P(\mathrm{M})$. For this purpose we need to specify all prior probabilities of the 'parent node' in the Bayesian network (i.e., X) and the conditional probabilities for the other 'child nodes', given the values of their parents.

Let us simplify our notation by using the following shorthand:

$$\begin{aligned} P(\mathrm{X}) &= x \qquad P(\mathrm{M}|\mathrm{X}) = m_x \\ P(\mathrm{A}|\mathrm{X}) &= a_x \qquad P(\mathrm{E}|\mathrm{A}) = e_a. \end{aligned}$$

The probabilities of the corresponding negated propositions are denoted with a bar, viz. $P(\mathrm{A}|\bar{\mathrm{X}}) = a_{\bar{x}}$, $P(\bar{\mathrm{A}}|\mathrm{X}) = \bar{a}_x$ and $P(\bar{\mathrm{A}}|\bar{\mathrm{X}}) = \bar{a}_{\bar{x}}$.

The first central assumption is that the prior probability of X lies in the open interval (0,1):

$$0 < x < 1. \tag{1}$$

The conditional probabilities are then constrained by the following assumptions:

$$m_x > m_{\bar{x}} \tag{2}$$
$$a_x > a_{\bar{x}} \tag{3}$$
$$e_a > e_{\bar{a}}. \tag{4}$$

The statements (1)-(3) encode probabilistically the concept of MEEGA since they allow for the possibility of a background assumption that supports both M and A. The statement (4) encodes probabilistically that the empirical evidence actually plays the role of evidence in favour of the model A.

With these assumptions one can prove (see Appendix A) the following theorem:

Theorem 1: $P(\mathrm{M}|\mathrm{E}) > P(\mathrm{M})$, if (1), (2), (3) and (4) are satisfied.

The satisfaction of Theorem 1 implies that E confirms M within a Bayesian analysis of confirmation. Within the framework of analogue simulation, provided we have some MEEGA with prior probability that is neither unity or zero, confirmation of a hypothesis regarding the target system can obtain based upon evidence relating to the source system. It is important to note again, that having independent grounds on which to support one of the common background assumptions will 'block' the inductive support E can give for M as that background assumption already has a large marginal probability. This does not pose a problem for this account but offers a way to distinguish between those circumstances in which the novel empirical evidence E can provide substantial inductive support for M and those circumstances it cannot be used for that purpose.

An important implication of the Bayesian analysis relates to the role of the syntactic isomorphism. The structure of the Bayesian network is such that the syntactic isomorphism is not explicitly represented. Furthermore, based upon the network, even if no syntactic isomorphism obtains between the modelling frameworks $\mathcal{M}$ and $\mathcal{A}$, one could sensibly talk about confirmation of M by E, provided there exists some non-empty set of shared background assumptions. The key point is that in such circumstances although confirmation of M would indeed obtain, there would be no 'analogue simulation'. As discussed above, the role of the isomorphism is to guarantee that there will be a term within the modelling language of $\mathcal{M}$ that is counterpart to the term within $\mathcal{A}$ that refers to $\mathcal{E}$. Without such a term within $\mathcal{M}$ there would be no sense in which $\mathcal{S}$ is acting as a simulator for the behaviour of $\mathcal{T}$. For analogue simulation with confirmation to obtain we require both MEEGA and a syntactic isomorphism.

To recapitulate, in this section we have demonstrated that confirmation via analogue simulation obtains within a Bayesian analysis provided there exists an inferential connection between the conditions of applicability of the target and system models. That is, if there exists a binary variable that is assumed to be positively correlated with the empirical adequacy of both the source and target models, then evidence in favour of the model of the source system can be used to make inferences about the target system. This, in-and-of-itself, is not a particularly surprising result, and certainly the demonstration of such in principle inferential relations is not an external validation of the framework for analogue simulation that is being proposed. Rather, we take the results of this section to: i) demonstrate the internal consistency of the informal arguments towards confirmation via analogue simulation given in (Dardashti et al. 2015); and ii) provide a powerful evaluative and heuristic tool for the analysis of analogue simulation as it exists within contemporary scientific practice. Two natural directions of further development are: i) the identification and evaluation of potential cases of MEEGA in other scientific examples (in addition to that considered in (Dardashti et al. 2015)); and ii) the refinement of the Bayesian model to include cases within more than one analogue system. The second of these will be pursued in the following section.

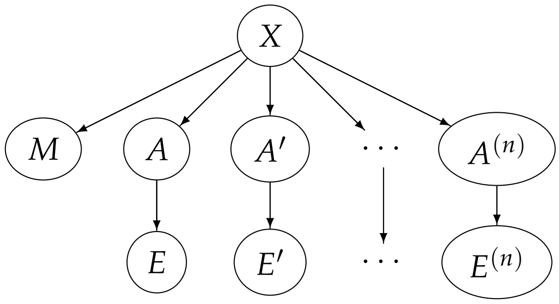

Figure 4: n-source system.

4 Multiple Sources

One important application of analogue simulation is in the context of universality arguments. In such cases the source system is 'multiply realisable' in that there are various different physical systems that can be used to implement the analogue simulation. Such a notion of 'multiply realisability' is intended to be something more than the variation of the material constitution of the source system. Such variation would involve keeping fixed the 'nomological behaviour' of the source system but changing the material constitution. Rather, the situation we are considering is when one varies the modelling frameworks used to construct the analogy, and in doing so considers equations that are syntactically isomorphic but extensionally distinct.

For example, consider again the dumb hole case. Rather than making use of the syntactic isomorphism between fluid mechanical and gravitational models we can draw inferences based upon analogue black holes constructed out of Bose-Einstein condensates, traveling refractive index interfaces in nonlinear optical media or 'slow light' in moving media (Carusotto et al. 2008). This is to vary both the material constitution and the nomological behaviour of the analogue system.

With such examples in mind, we can extend the analysis of the previous section to consider the case when we have multiple sources each providing independent evidence for the target system modelling framework. The expectation would be that adding more source systems should increase the degree of confirmation, but that this increase will eventually reach some 'saturation point'. This matches the intuition that, given some non-zero (or one) prior probability for the truth of the universality arguments, a small set of different successful realisations of the source system would be enough to provide strong evidence in favour of a hypotheses of regarding analogue behaviour in the target system.

Consider a Bayesian network for an n-source system (Figure 4). The question we would like to answer is how does the confirmation measure change as one increases the number of different analogue systems providing us with evidence. Following the same line of reasoning as the last section we assume:

$$a'_x > a'_{\bar{x}} \qquad e'_{a'} > e'_{\bar{a}'}. \tag{5}$$
$$a''_x > a''_{\bar{x}} \qquad e''_{a''} > e''_{\bar{a}''}. \tag{6}$$
$$\vdots$$
$$a^{(n)}_x > a^{(n)}_{\bar{x}} \qquad e^{(n)}_{a^{(n)}} > e^{(n)}_{\bar{a}^{(n)}}. \tag{7}$$

We can now calculate the difference measure of confirmation, which is defined as

$$\Delta^{(n)} := P(\mathrm{M}|\mathrm{E}, \mathrm{E}', ..., \mathrm{E}^{(n)}) - P(\mathrm{M}) \tag{8}$$

and obtain $\Delta^{(n)} > 0$ (see Appendix B).

It can further be shown that:

> **Theorem 2**: $\Delta^{(n)}$ is a strictly increasing function of the number of source systems.

This theorem implies that as the number of different analogue systems providing evidence increases so does the degree of confirmation.[5] Again, this is not a particularly surprising result. Given that confirmation via analogue simulation obtains for a single source system, one would expect that adding in more and more (independent) source systems would allow one to increase the degree of confirmation. The feature that is most interesting is not the fact that $\Delta^{(n)}$ is strictly increasing, but rather the functional form of this increase. In particular, the natural intuition is that as the number of source systems increases the increase the degree of confirmation would eventually saturate. One of the chief virtues of the Bayesian model for analogue simulation with multiple source systems is that it allows us to give an analytical expression for such a saturation point.

First, let us consider how $\Delta^{(n)}$ changes in the large n limit. A little analytical work (again see Appendix B) allows us to show that:

$$\lim_{n \to +\infty} \Delta^{(n)} \to \bar{x}(m_x - m_{\bar{x}}) = N_{\text{sat.}}. \tag{9}$$

This means that the maximum amount of confirmation one can obtain by adding in more and more sources is bounded by some finite number, $N_{\text{sat.}}$, determined by the prior probabilities $\bar{x}$, m_x and $m_{\bar{x}}$. Beyond this point, there is vanishingly small added value (in terms of confirmation) achieved by adding in more source systems. Two features of $N_{\text{sat.}}$ are worth remarking on. First, the higher the prior probability of X the lower the saturation point will be. This makes sense because the more sure we are of X to start with, the lower the limit on the extra information we can learn from $\mathrm{E}, \mathrm{E}', ..., \mathrm{E}^{(n)}$. Second, the higher the relative likelihood of M given X to M given ¬X (i.e. $m_x - m_{\bar{x}}$), the higher the saturation point. This makes sense because the stronger the relationship between X and M the more we can potentially learn from $\mathrm{E}, \mathrm{E}', ..., \mathrm{E}^{(n)}$.

[5] Theorem 2 does not depend on the choice of this particular confirmation measure and will also hold if we move to another confirmation measure (**?**).

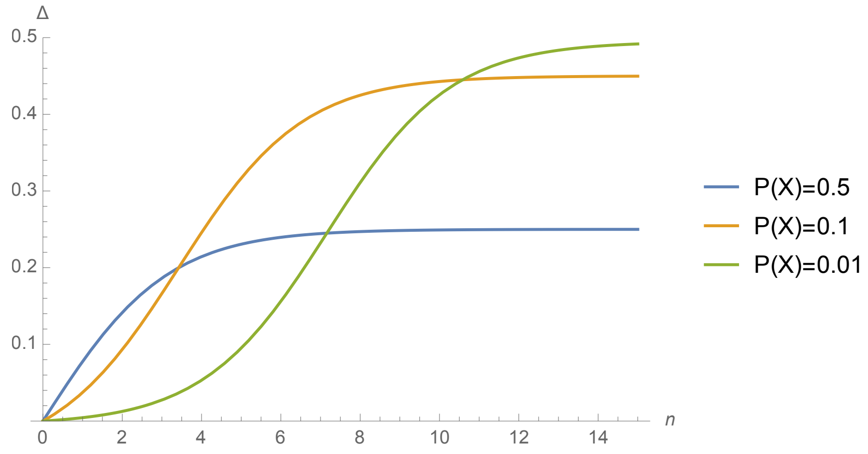

Figure 5: Confirmation measure dependence and saturation point.

A further interesting feature that we can examine is the speed with which the saturation point is approached. We can examine this 'rate of saturation' by plotting $\Delta^{(n)}$ for a set of prior probabilities of X.[6] As can be seen from Figure (5), the higher the prior probability of X, the quicker the saturation point is reached. Strikingly, for the values of the parameters considered, we find that given a prior of greater than 0.5 for X, saturation can be reached after only three or four successful analogue experiments.

This result is in tune with scientific intuitions regarding analogue simulation in the context of universality arguments. Consider, in particular the dumb hole Hawking radiation case. There has been, thus far, only one implementation of a source system that is reported to display the full Hawking effect: the Bose-Einstein condensate experiments of (Steinhauer 2014). Given initial confidence in the universality arguments, if another different implementation of a source system displaying the full Hawking effect was achieved, that should surely radically increase the belief in the astrophysical Hawking effect. However, once a few such examples were constructed, one would quickly stop gaining new insight. Conversely, given initial skepticism regarding the universality arguments, a second implementation of the dumb hole source system would not radically increase the belief in the astrophysical Hawking effect. Furthermore, in such circumstances it would only be after a diverse and extensive range of implementations of source systems that one would stop believing that new examples gave new information.

[6]See Equation (22) of Appendix B. Here we have assumed for simplicity that $\gamma^{(k)} = c$ for all k with $c > 1$. c measures both the likelihood of $\mathrm{A}^{(k)}$ given X and the likelihood of $\mathrm{E}^{(k)}$ given $\mathrm{A}^{(k)}$. The stronger the dependence of these the stronger the exponential increase of $\Delta^{(n)}$ with n. See

5 Conclusion

History is replete with examples of 'transformative' technology having a profound and lasting impact on the methodological foundations of science. Much recent literature in the philosophy of science has focused on the sense in which computer simulation should (or should not) be taken to have had such an impact.[7] Analogue simulation is a new inferential tool found at the cutting edge of modern science that we see good reasons to take as potentially transformative. Building upon the initial analysis of (Dardashti et al. 2015), in this paper we have applied a Bayesian analysis to explicate the structure of inferences that analogue simulation can and cannot allow us to make. Our two principal results are: i) that 'single source' confirmation via analogue simulation can obtain under certain conditions; and ii) that 'multiple source' confirmation via analogue simulation displays the generic feature of saturation in confirmatory power.

A Proof of Theorem 1

Let us start with the Bayesian network depicted in Figure 2. We have to show that

$$P(\mathrm{M}|\mathrm{E}) = \frac{P(\mathrm{M},\mathrm{E})}{P(\mathrm{E})} > P(\mathrm{M}). \tag{10}$$

The joint probability can be obtained in the following way[8]:

$$\begin{aligned} P(\mathrm{M},\mathrm{E}) &= \sum_{X,A} P(X,\mathrm{M},A,\mathrm{E}) \\ &= \sum_{X,A} P(X)P(\mathrm{M}|X)P(A|X)P(\mathrm{E}|A) \\ &= x m_x (a_x e_a + \bar{a}_x e_{\bar{a}}) + \bar{x} m_{\bar{x}} (a_{\bar{x}} e_a + \bar{a}_{\bar{x}} e_{\bar{a}}) \\ &= x m_x \alpha + \bar{x} m_{\bar{x}} \beta \end{aligned} \tag{11}$$

where we have defined

$$\alpha := a_x e_a + \bar{a}_x e_{\bar{a}} \tag{12}$$

$$\beta := a_{\bar{x}} e_a + \bar{a}_{\bar{x}} e_{\bar{a}}. \tag{13}$$

Similarly we obtain

$$\begin{aligned} P(\mathrm{E}) &= \sum_{X,A,M} P(X,M,A,\mathrm{E}) \\ &= x\alpha + \bar{x}\beta \end{aligned} \tag{14}$$

[7]See for example (Frigg and Reiss 2009; Parker 2009; Winsberg 2010; Beisbart and Norton 2012).

[8]See (Bovens and Hartmann 2004, Sect. 3.5) on the general methodology of reading joint probabilities from Bayesian networks.

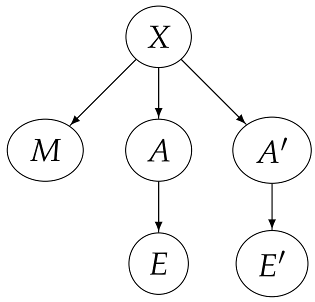

Figure 6: 2-source system.

and

$$P(\mathrm{M}) = xm_x + \bar{x}m_{\bar{x}}. \tag{15}$$

Defining the difference measure $\Delta := P(\mathrm{M}|\mathrm{E}) - P(\mathrm{M})$, we need to show that Δ is larger than zero. After some algebraic manipulation one obtains

$$\begin{aligned} \Delta &= \frac{xm_x\alpha + \bar{x}m_{\bar{x}}\beta - (xm_x + \bar{x}m_{\bar{x}})(x\alpha + \bar{x}\beta)}{x\alpha + \bar{x}\beta} \\ &= \frac{\bar{x}x(m_x - m_{\bar{x}})(\alpha - \beta)}{x\alpha + \bar{x}\beta}. \end{aligned} \tag{16}$$

It is easy to show that

$$\alpha - \beta = (a_x - a_{\bar{x}})(e_a - e_{\bar{a}}) \tag{17}$$

so it follows that

$$\Delta = \frac{\bar{x}x(m_x - m_{\bar{x}})(a_x - a_{\bar{x}})(e_a - e_{\bar{a}})}{x\alpha + \bar{x}\beta}. \tag{18}$$

So if (2), (3) and (4) are satisfied it follows that $\Delta > 0$, which needed to be shown.

B Proofs for n Source Systems

To see how the previous theorem can be generalized to the n source systems represented in Figure 4 let us consider first the 2-source system represented in Figure 6.

We need to show that $P(\mathrm{M}|\mathrm{E},\mathrm{E}') = P(\mathrm{M},\mathrm{E},\mathrm{E}')/P(\mathrm{E},\mathrm{E}') > P(\mathrm{M})$. Let us start with the following joint probability

$$\begin{aligned} P(\mathrm{M},\mathrm{E},\mathrm{E}') &= \sum_{X,A,A'} P(X,\mathrm{M},A,\mathrm{E},A',\mathrm{E}') \\ &= \sum_{\mathrm{X},\mathrm{A},\mathrm{A}'} P(\mathrm{X})P(\mathrm{M}|\mathrm{X})P(\mathrm{A}|\mathrm{X})P(\mathrm{E}|\mathrm{A})P(\mathrm{A}'|\mathrm{X})P(\mathrm{E}'|\mathrm{A}') \\ &= xm_x\alpha\alpha' + \bar{x}m_{\bar{x}}\beta\beta' \end{aligned} \tag{19}$$

where α' and β' is defined identically to (12) and (13) with e and a replaced with e' and a'.
Similarly we obtain

$$\begin{aligned} P(\mathrm{E},\mathrm{E}') &= \sum_{X,A,A'} P(X)P(\mathrm{E}|A)P(A|\mathrm{X})P(\mathrm{E}'|A')P(A'|\mathrm{X}) \\ &= x\alpha\alpha' + \bar{x}\beta\beta'. \end{aligned} \tag{20}$$

Defining as before $\Delta' := P(\mathrm{M}|\mathrm{E},\mathrm{E}') - P(\mathrm{M})$ it follows that

$$\Delta' = \frac{x\bar{x}(m_x - m_{\bar{x}})(\alpha\alpha' - \beta\beta')}{x\alpha\alpha' + \bar{x}\beta\beta'}. \tag{21}$$

Now $\alpha\alpha' - \beta\beta'$ is larger than zero iff $(\alpha\beta)/(\alpha'\beta') > 1$. This in turn is the case when $\alpha - \beta > 0$ and $\alpha' - \beta' > 0$. Both of these conditions are satisfied due to (3)-(5). So it follows that (21) is larger than zero.
It is now straightforward to generalize to the n-source system represented in Figure 4. For the n-source system we need to show that $\Delta^{(n)} = P(\mathrm{M}|\mathrm{E},\mathrm{E}',...,\mathrm{E}^{(n)}) - P(\mathrm{M}) > 0$. It follows from the above consideration that

$$\Delta^{(n)} = \frac{x\bar{x}(m_x - m_{\bar{x}})(\prod_{k=0}^{n}\alpha^{(k)} - \prod_{k=0}^{n}\beta^{(k)})}{x\prod_{k=0}^{n}\alpha^{(k)} + \bar{x}\prod_{k=0}^{n}\beta^{(k)}} \tag{22}$$

with $\alpha^{(0)} = \alpha$ and $\beta^{(0)} = \beta$. We have again $\Delta^{(n)} > 0$ once (3)-(7) is satisfied.

Let us define $\gamma^{(n)} := \prod_{k=0}^{n}\alpha^{(k)}/\beta^{(k)}$. Since $\alpha^{(k)} > \beta^{(k)}$ for all k, $\gamma^{(n)}$ increases as n increases. Furthermore, we have

$$\frac{\partial\Delta^{(n)}}{\partial\gamma^{(n)}} = \frac{x\bar{x}(m_x - m_{\bar{x}})}{(x\gamma^{(n)} + \bar{x})^2} > 0. \tag{23}$$

So as n increases, i.e. as the number of analogue systems providing evidence increases, so does the amount of confirmation. Setting $\kappa := x\bar{x}(m_x - m_{\bar{x}})$ we obtain the large n behaviour:

$$\lim_{n\to+\infty}\Delta^{(n)} = \lim_{x\to+\infty}\kappa\frac{\gamma^{(n)} - 1}{x\gamma^{(n)} + \bar{x}} \to \frac{\kappa}{x}. \tag{24}$$

References

Bailer-Jones, D. M. (2009). *Scientific Models in Philosophy of Science*. Pittsburgh: University of Pittsburgh Press.

Bartha, P. (2013). 'Analogy and analogical reasoning'. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Fall 2013 ed.).

Bartha, P. F. (2010). *By Parallel Reasoning: The Construction and Evaluation of Analogical Arguments*. Oxford: Oxford University Press.

Batterman, R. W. (2000). 'Multiple realizability and universality'. *The British Journal for the Philosophy of Science* ***51***(1), pp. 115–45.

Batterman, R. W. (2002). *The devil in the details: Asymptotic reasoning in explanation, reduction, and emergence*. Oxford: Oxford University Press.

Beisbart, C. and J. D. Norton (2012). 'Why Monte Carlo Simulations Are Inferences and Not Experiments'. *International Studies in the Philosophy of Science 26*(4), 403–22.

Betz, G. (2013). 'Revamping hypothetico-deductivism: A dialectic account of confirmation'. *Erkenntnis 78*(5), 991–1009.

Bovens, L. and S. Hartmann (2004). *Bayesian Epistemology*. Oxford: Oxford University Press.

Carusotto, I., S. Fagnocchi, A. Recati, R. Balbinot, and A. Fabbri (2008). 'Numerical observation of Hawking radiation from acoustic black holes in atomic Bose–Einstein condensates'. *New Journal of Physics 10*(10), 103001.

Dardashti, R., , K. Thébault, and E. Winsberg (2015). 'Confirmation via analogue simulation: What dumb holes could tell us about gravity'. *The British Journal for the Philosophy of Science (forthcoming)*.

Frigg, R. and J. Reiss (2009). 'The philosophy of simulation: hot new issues or same old stew?'. *Synthese 169*(3), 593–613.

Gerritsma, R., G. Kirchmair, F. Zähringer, E. Solano, R. Blatt, and C. Roos (2010). 'Quantum simulation of the Dirac equation'. *Nature 463*(7277), 68–71.

Hartmann, S. and J. Sprenger (2010). 'Bayesian epistemology'. In S. Bernecker and D. Pritchard (Eds.), *Routledge Companion to Epistemology*, pp. 609–620. London: Routledge.

Hawking, S. W. (1975). 'Particle creation by black holes'. *Communications in mathematical physics 43*(3), 199–220.

Hesse, M. (1964). 'Analogy and confirmation theory'. *Philosophy of Science 31*(4), 319–27.

Hesse, M. B. (1966). *Models and Analogies in Science*, Volume 7. Notre Dame: University of Notre Dame Press.

Hume, D. (1738/1978). *A Treatise of Human Nature*. Oxford: Oxford University Press.

Keynes, J. M. (1921). *A Treatise on Probability*. London: Macmillan & Co.

Logan, J. (1962). 'Hydrodynamic analog of the classical field equations'. *Physics of Fluids (1958-1988) 5*(7), 868–869.

Norton, J. D. (2011). 'Analogy'. *Manuscript, http://www. pitt. edu/~ jdnorton/papers/material_theory/Analogy. pdf*.

Parker, W. S. (2009). 'Does matter really matter? Computer simulations, experiments, and materiality'. *Synthese 169*(3), 483–96.

Reid, T. and W. Hamilton (1850). *Essays on the Intellectual Powers of Man*. Cambridge: J. Bartlett.

Salmon, W. (1990). 'Rationality and objectivity in science or Tom Kuhn meets Tom Bayes'. In C. Wade Savage (Ed.), *Scientific theories*, Volume 14, pp. 175–204. Minnesota: University of Minnesota Press.

Steinhauer, J. (2014). 'Observation of self-amplifying hawking radiation in an analogue black-hole laser'. *Nature Physics 10*, 864–869.

Unruh, W. (1981). 'Experimental black-hole evaporation?'. *Physical Review Letters 46*(21), 1351–53.

Unruh, W. G. (2014). Has hawking radiation been measured? *Foundations of Physics 44*(5), 532–545.

Unruh, W. G. and R. Schützhold (2005). 'Universality of the Hawking effect'. *Physical Review D 71*(2), 024028.

Winsberg, E. (2010). *Science in the Age of Computer Simulation*. Chicago: University of Chicago Press.