

An Ontological Interpretation of the Wave Function

Shan Gao*

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Abstract

It is argued that, based on a new analysis of two-body systems, wave function realism seems to imply a unique ontological interpretation of the wave function, according to which the wave function represents the state of random discontinuous motion of particles, and in particular, its modulus square gives the probability density of the particles appearing in certain positions in space.

The wavefunction gives not the density of stuff, but gives rather (on squaring its modulus) the density of probability. Probability of what exactly? Not of the electron being there, but of the electron being found there, if its position is 'measured'. Why this aversion to 'being' and insistence on 'finding'? The founding fathers were unable to form a clear picture of things on the remote atomic scale. (Bell 1990)

1 Introduction

The physical meaning of the wave function is an important interpretative problem of quantum mechanics. Notwithstanding more than eighty years' developments of the theory, it is still an unsolved issue. During recent years, more and more authors have done research on the ontological meaning of the wave function (see, e.g. Monton 2002; Lewis 2004; Gao 2011a, 2011b; Pusey, Barrett and Rudolph 2012; Ney and Albert 2013). In particular, Pusey, Barrett and Rudolph (2012) demonstrated that, under an independence assumption, the wave function of a quantum system is a representation of the physical state of the system. This poses a further question, namely whether wave function realism can be argued without resorting to nontrivial assumptions such as the independence assumption (cf. Lewis et al 2012).

*Institute for the History of Natural Sciences, Chinese Academy of Sciences, Beijing 100190, P. R. China. E-mail: gaoshan@ihns.ac.cn.

Moreover, a harder problem is to determine the ontological meaning of the wave function, which is also a hot topic of debate in the alternatives to quantum mechanics such as the de Broglie-Bohm theory (Belot 2012).

In this paper, we will introduce an argument for the reality of the wave function in terms of protective measurements, which does not depend on nontrivial assumptions. Moreover, we will argue that, based on a new analysis of two-body systems, wave function realism seems to imply an unique ontological interpretation of the wave function, according to which the wave function represents the state of random discontinuous motion of particles, and in particular, its modulus square gives the probability density of the particles appearing in certain positions in space.

2 On the reality of the wave function

The meaning of the wave function in quantum mechanics is usually analyzed in the context of conventional impulsive measurements. Although the wave function of a quantum system is in general extended over space, an ideal position measurement will inevitably collapse the wave function and can only detect the system in a random position in space. Thus it seems natural to assume that the wave function is only related to the probabilities of these random measurement results as in the standard probability interpretation. However, it has been known that the wave function of a single quantum system can be protectively measured (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009)¹. During a protective measurement, the measured state is protected by an appropriate procedure (e.g. via the quantum Zeno effect) so that it neither changes nor becomes entangled with the state of the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system, even if the system is initially not in an eigenstate of the measured observable, and in particular, the wave function of the system can also be measured as expectation values of certain observables.

It can be argued that protective measurements provide a strong support for wave function realism, according to which the wave function of a quantum system represents the physical state of the system or the state of a physical entity². The argument is as follows. According to quantum mechanics,

¹It can be expected that protective measurement will be realized in the near future with the rapid development of quantum technologies.

²Note that several authors, including the inventors of protective measurements, have analyzed the implications of protective measurements for the ontological status of the wave function (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Anandan 1993; Dickson 1995). However, their arguments seem to rely on the presupposition that protective measurements are completely reliable, which is not true (Dass and Qureshi 1999; Vaidman 2009; Gao 2013a). A realistic protective measurement can never be performed

we can prepare a single measured system whose wave function is $\psi(t)$ at a given instant t . For example, the measured system is an electron being in the ground state of a Hydrogen atom. Now, by a protective measurement, we can obtain the expectation value of the measured observable in this state without disturbing the state (though with probability smaller than one in realistic situations)³. Moreover, by a series of protective measurements of certain observables, we can obtain the value of $\psi(t)$ *only* from this measured system. Thus we can reach the conclusion that the expectation values of observables are the physical properties of a single quantum system, and the wave function of the system represents the physical property of the system⁴. In particular, $\psi(x, t)$, the spatial wave function of the system in position x at instant t , represents the physical property of the system in position x at instant t . This also means that for a quantum system, there is a physical entity spreading out over a region of space where the spatial wave function of the system is not zero.

Here we assume a realist view on the theory-reality relation, which means that the theoretical terms expressed in the language of mathematics connect to the entities existing in the physical world. On this view, the wave function in quantum mechanics describes either the state of an ensemble of identical systems or the state of a single system. Since we can measure the wave function only from a single system by protective measurements, the wave function must represent the property of a single system⁵. Note that this

on a single quantum system with absolute certainty. For example, for a realistic protective measurement of an observable A in a non-degenerate energy eigenstate whose measuring interval T is finite, there is always a tiny probability proportional to $1/T^2$ to obtain a different result $\langle A \rangle_{\perp}$, where \perp refers to a normalized state in the subspace normal to the measured state as picked out by the first order perturbation theory. The following argument will not depend on this presupposition.

³When the measurement obtains the expectation value of the measured observable in the measured state, the measured state is not changed. Moreover, the probability of obtaining a different result and collapsing the measured state can be made arbitrarily small in principle. By comparison, the eigenvalues values of the measured observable being measurement results are only consequences of non-protective, strong measurements, which disturb the measured state strongly and are arguably not good, qualified measurements.

⁴There might also exist other components of the underlying physical state, which are not measurable by protective measurements and not described by the wave function, e.g. the positions of the Bohmian particles in the de Broglie-Bohm theory. In this case, however, the wave function is still uniquely determined by the underlying physical state, though it is not a complete representation of the physical state. As a result, the epistemic interpretation of the wave function will be ruled out (cf. Lewis et al 2012). Certainly, the wave function also plays an epistemic role by giving the probability distribution of the results of conventional impulsive measurements according to the Born rule. However, this role is secondary and determined by the complete quantum dynamics that describes the measuring process, e.g. the collapse dynamics in dynamical collapse theories.

⁵We can also give a PBR-like argument for ψ -ontology in terms of protective measurements (cf. Pusey, Barrett and Rudolph 2012). For two (known) nonorthogonal states of a quantum system, the results of the protective measurements of them may be different with probability that can be arbitrarily close to one. If there exists a finite probability that

conclusion is independent of whether the wave function of the measured system is known beforehand for protective measurements. Even though we know the wave function, which is an abstract mathematical object, we still don't know its physical meaning, while protective measurements can help answer this fundamental question of quantum mechanics⁶.

3 Analysis of a two-body system

In the following, we will further analyze the ontological meaning of the wave function. Consider a two-body system whose wave function is defined in a six-dimensional configuration space. We first suppose the wave function of the system is localized in one position $(x_1, y_1, z_1, x_2, y_2, z_2)$ in the configuration space of the system at a given instant. This wave function can be decomposed into a product of two wave functions which are localized in positions (x_1, y_1, z_1) and (x_2, y_2, z_2) in our ordinary three-dimensional space, respectively.

According to wave function realism, the wave function of a quantum system represents the property or state of a physical entity, which further means that the entity exists in the region of space where its wave function is not zero. Therefore, when assuming wave function realism, the above wave function describes two independent physical entities, which are localized in positions (x_1, y_1, z_1) and (x_2, y_2, z_2) in our three-dimensional space, respectively. Moreover, the Schrödinger equation that governs the evolution of the system may further indicate that these two physical entities have masses m_1 and m_2 (as well as charges Q_1 and Q_2 etc), respectively. Note that these properties are instantaneous properties that are input to the Schrödinger equation, not generated by time evolution represented by the equation, and these properties having two different values also indicate that there are two different physical entities.

Now suppose the wave function of the two-body system is localized in two positions $(x_1, y_1, z_1, x_2, y_2, z_2)$ and $(x'_1, y'_1, z'_1, x'_2, y'_2, z'_2)$ in the configuration space of the system at a given instant⁷. According to the above analysis,

these two nonorthogonal states correspond to the same physical state λ , then when assuming λ determines the probability of measurement results as the PBR theorem assumes, the results for the two nonorthogonal states will be the same with the finite probability. This leads to a contradiction. This argument only considers one quantum system, and avoids the independence assumption used by the PBR theorem.

⁶In addition, as pointed out by Aharonov, Anandan and Vaidman (1996), the wave function of the measured system may be unknown beforehand when splitting the procedure of a protective measurement into two stages. The first is a protection, made by one experimenter or even just by nature, and the second is performed by another experimenter who does not know the measured state. What this experimenter needs to know is that the state is protected and what is the degree of protection, and he does obtain new information by protective measurement.

⁷This is a so-called entangled state, which can be generated from a non-entangled

when assuming wave function realism, the wave function of the two-body system being localized in position $(x_1, y_1, z_1, x_2, y_2, z_2)$ in configuration space means that physical entity 1 with mass m_1 and charge Q_1 exists in position (x_1, y_1, z_1) in three-dimensional space, and physical entity 2 with mass m_2 and charge Q_2 exists in position (x_2, y_2, z_2) in three-dimensional space. Similarly, the wave function of the two-body system being localized in position $(x'_1, y'_1, z'_1, x'_2, y'_2, z'_2)$ in configuration space means that physical entity 1 exists in position (x'_1, y'_1, z'_1) in three-dimensional space, and physical entity 2 exists in position (x'_2, y'_2, z'_2) in three-dimensional space. Moreover, according to wave function realism, the wave function of the two-body system being localized in both positions $(x_1, y_1, z_1, x_2, y_2, z_2)$ and $(x'_1, y'_1, z'_1, x'_2, y'_2, z'_2)$ in configuration space means that the above two situations both exist in reality⁸. The question is: In what form?

An obvious existent form is that physical entity 1 exists in both positions (x_1, y_1, z_1) and (x'_1, y'_1, z'_1) , and physical entity 2 exists in both positions (x_2, y_2, z_2) and (x'_2, y'_2, z'_2) . However, wave function realism also requires that a physical entity described by its wave function does not exist in the region of space where the wave function is zero. Therefore, when physical entity 1 exists in (x_1, y_1, z_1) , physical entity 2 cannot exist in (x'_2, y'_2, z'_2) , and when physical entity 1 exists in (x'_1, y'_1, z'_1) , physical entity 2 cannot exist in (x_2, y_2, z_2) , or vice versa. In other words, the wave function that describes this existent form should be localized in four positions $(x_1, y_1, z_1, x_2, y_2, z_2)$, $(x'_1, y'_1, z'_1, x'_2, y'_2, z'_2)$, $(x_1, y_1, z_1, x'_2, y'_2, z'_2)$, and $(x'_1, y'_1, z'_1, x_2, y_2, z_2)$ in the configuration space of the system. Therefore, the above existent form, which seems to be the only possible form, is not possible.

It seems that there is a contradiction here, and anti-realists may readily welcome this result as a no-go result for wave function realism. However, wave function realism is not dead; there is still one possibility, though which is hardly imaginable. It can be seen that the contradiction only requires that the above two situations cannot exist at the same time at a single instant. As we will show below, however, they may exist “at the same time” during an infinitesimal time interval, in a way of time division. (This means that the state of the physical entity described by the wave function is defined during an infinitesimal time interval. Since there is no physical difference between an instant and an infinitesimal time interval, this does not influence the predictions of quantum mechanics and is still consistent with experiments.)

Concretely speaking, the situation in which physical entity 1 is in (x_1, y_1, z_1) and physical entity 2 is in (x_2, y_2, z_2) exists in one part of continuous time,

state by the time evolution of the system. The existence of entangled states have also been confirmed by experiments.

⁸Note again that a physical entity exists in the region of space where its wave function is not zero. Moreover, for the system being in this superposition state there are still two different physical entities, as the properties such as mass and charge do not change, and the Schrödinger equation that governs the evolution of the system is also the same.

and the situation in which physical entity 1 is in (x'_1, y'_1, z'_1) and physical entity 2 is in (x'_2, y'_2, z'_2) exists in the other part. The restriction is that the temporal part in which each situation exists cannot be a continuous time interval during an arbitrarily short time interval; otherwise the wave function describing the state in the time interval will be not the original superposition of two branches, but one of the branches, according to wave function realism. This means that the set of the instants when each situation exists is not a continuous set but a discontinuous, dense set. At some discontinuous instants, physical entity 1 with mass m_1 and charge Q_1 exists in position (x_1, y_1, z_1) , and physical entity 2 with mass m_2 and charge Q_2 exists in position (x_2, y_2, z_2) , and at other discontinuous instants, physical entity 1 exists in position (x'_1, y'_1, z'_1) , and physical entity 2 exists in position (x'_2, y'_2, z'_2) . By this way of time division, the above two situations exist “at the same time” during an arbitrarily short time interval or during an infinitesimal time interval.

This way of time division also implies a strange picture of motion for the involved physical entities. It is as follows. Physical entity 1 with mass m_1 and charge Q_1 jumps discontinuously between positions (x_1, y_1, z_1) and (x'_1, y'_1, z'_1) , and physical entity 2 with mass m_2 and charge Q_2 jumps discontinuously between positions (x_2, y_2, z_2) and (x'_2, y'_2, z'_2) . Moreover, they jump in a precisely simultaneous way. When physical entity 1 jumps from position (x_1, y_1, z_1) to position (x'_1, y'_1, z'_1) , physical entity 2 jumps from position (x_2, y_2, z_2) to position (x'_2, y'_2, z'_2) , or vice versa. In the limit situation where position (x_2, y_2, z_2) is the same as position (x'_2, y'_2, z'_2) , physical entities 1 and 2 are no longer entangled, while physical entity 1 with mass m_1 and charge Q_1 still jumps discontinuously between positions (x_1, y_1, z_1) and (x'_1, y'_1, z'_1) . This means that the picture of discontinuous motion also exists for one-body systems. Since quantum mechanics does not provide further information about the positions of physical entities at each instant, the discontinuous motion described by the theory is also essentially random.

The above analysis can be extended to an arbitrary entangled wave function for a N-body system. Since each physical entity is only in one position in space at each instant⁹, it may well be called particle. Here the concept of particle is used in its usual sense. A particle is a small localized object with mass and charge etc, and it is only in one position in space at an instant. Therefore, wave function realism seems to require that the physical entities described by the wave function such as physical entities 1 and 2 are localized particles. Moreover, the motion of these particles is not continuous but discontinuous and random in nature, and especially, the motion of entangled particles is precisely simultaneous. In the next section, we will further analyze the relationship between random discontinuous motion of particles

⁹When considering interactions, it can be further argued that the mass and charge of the entity are also localized in the position in efficiency.

and the wave function.

4 The wave function as a description of random discontinuous motion of particles

In this section, we will give a more detailed analysis of random discontinuous motion and the meaning of the wave function (Gao 1993, 1999, 2000, 2003, 2006, 2008, 2011a, 2011b, 2013b).

4.1 An analysis of random discontinuous motion of particles

In the following, we will give a strict description of random discontinuous motion of particles based on measure theory. For simplicity but without losing generality, we will mainly analyze the one-dimensional motion that corresponds to the point set in two-dimensional space and time. The results can be readily extended to the three-dimensional situation.

We first analyze the random discontinuous motion of a single particle. From a logical point of view, the particle must have an instantaneous property (as a probabilistic instantaneous condition) that determines the probability density for it to appear in every position in space; otherwise the particle would not “know” how frequently it should appear in each position in space. This property is usually called indeterministic disposition or propensity in the literature, and it can be represented by $\varrho(x, t)$, which satisfies the nonnegative condition $\varrho(x, t) \geq 0$ and the normalization relation $\int_{-\infty}^{+\infty} \varrho(x, t) dx = 1$. We suppose the disposition function $\varrho(x, t)$ is differentiable with respect to both x and t .

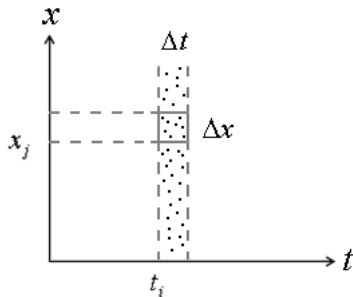


Fig.1 The description of random discontinuous motion of a single particle

Now consider the state of motion of the particle in finite intervals Δt and Δx near a space-time point (t_i, x_j) as shown in Fig. 1. The positions of the particle form a random, discontinuous trajectory in this square region¹⁰. We

¹⁰Recall that a trajectory function $x(t)$ is essentially discontinuous if it is not continuous at every instant t . A trajectory function $x(t)$ is continuous if and only if for every t and every real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that whenever a point t_0

study the projection of this trajectory in the t -axis, which is a dense instant set in the time interval Δt . Let W be the discontinuous trajectory of the particle and Q be the square region $[x_j, x_j + \Delta x] \times [t_i, t_i + \Delta t]$. The dense instant set can be denoted by $\pi_t(W \cap Q) \in \mathfrak{R}$, where π_t is the projection on the t -axis. According to the measure theory, we can define the Lebesgue measure:

$$M_{\Delta x, \Delta t}(x_j, t_i) = \int_{\pi_t(W \cap Q) \in \mathfrak{R}} dt. \quad (1)$$

Since the sum of the measures of all such dense instant sets in the time interval Δt is equal to the length of the continuous time interval Δt , we have:

$$\sum_j M_{\Delta x, \Delta t}(x_j, t_i) = \Delta t. \quad (2)$$

Then we can define the measure density as follows:

$$\rho(x, t) = \lim_{\Delta x, \Delta t \rightarrow 0} M_{\Delta x, \Delta t}(x, t) / (\Delta x \cdot \Delta t). \quad (3)$$

This quantity provides a strict description of the position distribution of the particle or the relative frequency of the particle appearing in an infinitesimal space interval dx near position x during an infinitesimal interval dt near instant t , and it satisfies the normalization relation $\int_{-\infty}^{+\infty} \rho(x, t) dx = 1$ by Eq. (2). Note that the existence of the limit relies on the continuity of the evolution of $\rho(x, t)$, the property of the particle that determines the probability density of it appearing in every position in space. In fact, $\rho(x, t)$ is determined by $\varrho(x, t)$, and there exists the relation $\rho(x, t) = \varrho(x, t)$. We call $\rho(x, t)$ position measure density or position density in brief.

Since the position density $\rho(x, t)$ changes with time in general, we may further define the position flux density $j(x, t)$ through the relation $j(x, t) = \rho(x, t)v(x, t)$, where $v(x, t)$ is the velocity of the local position density. It describes the change rate of the position density. Due to the conservation of measure, $\rho(x, t)$ and $j(x, t)$ satisfy the continuity equation:

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial j(x, t)}{\partial x} = 0. \quad (4)$$

The position density $\rho(x, t)$ and position flux density $j(x, t)$ provide a complete description of the state of random discontinuous motion of a single particle (during an infinitesimal time interval).

The description of the motion of a single particle can be extended to the motion of many particles. At each instant a quantum system of N particles can be represented by a point in an $3N$ -dimensional configuration space.

has distance less than δ to t , the point $x(t_0)$ has distance less than ε to $x(t)$.

During an infinitesimal time interval, these particles perform random discontinuous motion in the real space, and correspondingly, this point performs random discontinuous motion in the configuration space and forms a cloud there. Then, similar to the single particle case, the state of the system can be represented by the joint position density $\rho(x_1, x_2, \dots, x_N, t)$ and joint position flux density $j(x_1, x_2, \dots, x_N, t)$ defined in the configuration space. They also satisfy the continuity equation:

$$\frac{\partial \rho(x_1, x_2, \dots, x_N, t)}{\partial t} + \sum_{i=1}^N \frac{\partial j(x_1, x_2, \dots, x_N, t)}{\partial x_i} = 0. \quad (5)$$

The joint position density $\rho(x_1, x_2, \dots, x_N, t)$ represents the probability density that particle 1 appears in position x_1 and particle 2 appears in position x_2 , ..., and particle N appears in position x_N ¹¹.

4.2 Interpreting the wave function

Although the motion of particles is essentially discontinuous and random, the discontinuity and randomness of motion are absorbed into the state of motion, which is defined during an infinitesimal time interval and represented by the position density $\rho(x, t)$ and position flux density $j(x, t)$. Therefore, the evolution of the state of random discontinuous motion of particles may obey a deterministic continuous equation. By considering the continuity equation and assuming that the nonrelativistic equation of random discontinuous motion is the Schrödinger equation, it can be shown that both $\rho(x, t)$ and $j(x, t)$ can be expressed by the wave function in a unique way¹²:

$$\rho(x, t) = |\psi(x, t)|^2, \quad (6)$$

$$j(x, t) = \frac{\hbar}{2mi} [\psi^*(x, t) \frac{\partial \psi(x, t)}{\partial x} - \psi(x, t) \frac{\partial \psi^*(x, t)}{\partial x}]. \quad (7)$$

Note that the relation between $j(x, t)$ and $\psi(x, t)$ depends on a concrete evolution under an external potential such as electromagnetic vector potential.

¹¹When these N particles are independent, the density $\rho(x_1, x_2, \dots, x_N, t)$ can be reduced to the direct product of the density for each particle, namely $\rho(x_1, x_2, \dots, x_N, t) = \prod_{i=1}^N \rho(x_i, t)$.

¹²Here is a brief explanation of how to derive the relation between $\rho(x, t)$ and $\psi(x, t)$. Consider a very simple case where the wave function of a quantum system is localized in two positions with different weights $|\psi_1|^2$ and $|\psi_2|^2$, which satisfy the normalization relation $|\psi_1|^2 + |\psi_2|^2 = 1$. For this case, the position densities in these two position also satisfy the normalization relation $\rho_1 + \rho_2 = 1$. Suppose the relation between $|\psi|^2$ and ρ , j is $|\psi|^2 = f(\rho, j)$. Then we have $|\psi_1|^2 = f(\rho_1, j)$ and $|\psi_2|^2 = f(\rho_2, j)$. Using the relation $\rho_1 + \rho_2 = 1$, the relation $|\psi_1|^2 + |\psi_2|^2 = 1$ becomes $f(\rho_1, j) + f(1 - \rho_1, j) = 1$. That this equation holds true for any ρ_1 and j requires $f(\rho) = \rho$. Note that if $f(\rho) \neq \rho$ is an n -order power function of ρ , then the equation can only have n solutions in terms of ρ .

By contrast, the relation $\rho(x, t) = |\psi(x, t)|^2$ holds true universally, independently of concrete evolution. Correspondingly, the wave function $\psi(x, t)$ can be uniquely expressed by $\rho(x, t)$ and $j(x, t)$ (except for a constant phase factor):

$$\psi(x, t) = \sqrt{\rho(x, t)} e^{im \int_{-\infty}^x \frac{j(x', t)}{\rho(x', t)} dx' / \hbar}. \quad (8)$$

In this way, the wave function $\psi(x, t)$ also provides a complete description of the state of random discontinuous motion of particles. For the motion of many particles, the joint position density and joint position flux density are defined in the 3N-dimensional configuration space, and thus the many-particle wave function, which is composed of these two quantities, is also defined in the 3N-dimensional configuration space.

One important point needs to be stressed here. Since the wave function in quantum mechanics is defined at a given instant, not during an infinitesimal time interval, it should be regarded not simply as a description of the state of motion of particles, but more suitably as a description of the dispositional property of the particles that determines their random discontinuous motion at a deeper level¹³. In particular, the modulus squared of the wave function determines the probability density that the particles appear in certain positions in space. By contrast, the density and flux density of the particle cloud in configuration space, which are defined during an infinitesimal time interval, are only a description of the state of the resulting random discontinuous motion of particles, and they are determined by the wave function. In this sense, we may say that the motion of particles is “guided” by their wave function in a probabilistic way.

4.3 On momentum, energy and spin

We have been discussing random discontinuous motion of particles in real space. Does the picture of random discontinuous motion exist for other dynamical variables such as momentum and energy? Since there are also wave functions of these variables in quantum mechanics, it seems tempting to assume that the above interpretation of the wave function in position space also applies to the wave functions in momentum space etc¹⁴. This means that when a particle is in a superposition of the eigenstates of a variable, it also undergoes random discontinuous motion among the corresponding eigenvalues of this variable. For example, a particle in a superposition of energy eigenstates also undergoes random discontinuous motion among all energy eigenvalues. At each instant the energy of the particle is definite,

¹³For a many-particle system in an entangled state, this dispositional property is possessed by the whole system.

¹⁴Under this assumption, the ontology of the theory will not only include the wavefunction and the particle position, but also include momentum and energy.

randomly assuming one of the energy eigenvalues with probability given by the modulus squared of the wave function at this energy eigenvalue, and during an infinitesimal time interval the energy of the particle spreads throughout all energy eigenvalues. Since the values of two noncommutative variables (e.g. position and momentum) at every instant may be mutually independent, the objective value distribution of every variable can be equal to the modulus squared of its wave function and consistent with quantum mechanics¹⁵.

However, there is also another possibility, namely that the picture of random discontinuous motion exists only for position, while momentum, energy etc do not undergo random discontinuous change among their eigenvalues. This is a minimum formulation in the sense that the ontology of the theory only includes the wave function and the particle position. On this view, the position of a particle is an instantaneous property of the particle defined at instants, while momentum and energy are properties relating only to its state of motion (e.g. momentum and energy eigenstates), which is formed by the motion of the particle during an infinitesimal time interval¹⁶. This may avoid the problem of defining the momentum and energy of a particle at instants. Certainly, we can still talk about momentum and energy on this view. For example, when a particle is in an eigenstate of the momentum or energy operator, we can say that the particle has definite momentum or energy, whose value is the corresponding eigenvalue. Moreover, when a particle is in a momentum or energy superposition state and the momentum or energy branches are well separated in space, we can still say that the particle has definite momentum or energy in certain local regions.

Lastly, we note that spin is a more distinct property. Since the spin of a free particle is always definite along one direction, the spin of the particle does not undergo random discontinuous motion, though a spin eigenstate along one direction can always be decomposed into two different spin eigenstates along another direction. But if the spin state of a particle is entangled with its spatial state due to interaction and the branches of the entangled state are well separated in space, the particle in different branches will have different spin, and it will also undergo random discontinuous motion be-

¹⁵Note that for random discontinuous motion a property (e.g. position) of a quantum system in a superposed state of the property is indeterminate in the sense of usual hidden variables, though it does have a definite value at each instant. For this reason, the particle position should not be called hidden variable for random discontinuous motion of particles, and the resulting theory is not a hidden variable theory either. This makes the theorems that restrict hidden variables such as the Kochen-Specker theorem irrelevant. Another way to see this is to realize that random discontinuous motion of particles alone does not provide a way to solve the measurement problem, and wavefunction collapse may also be needed. For details see Gao (2013b).

¹⁶It is worth stressing that the particle position here is different from the position property described by the position operator in quantum mechanics, and the latter is also a property relating only to the state of motion of the particle such as position eigenstates.

tween these different spin states. This is the situation that usually happens during a spin measurement.

5 Conclusions

Based on a new ontological analysis of two-body systems, we argue that wave function realism seems to imply that microscopic particles such as electrons are still particles, but they move in a discontinuous and random way. Moreover, the wave function describes the state of random discontinuous motion of particles, and at a deeper level, it represents the dispositional property of the particles that determines their random discontinuous motion. In this way, quantum mechanics, like Newtonian mechanics, also deals with the motion of particles in space and time, and it is essentially a physical theory about the laws of random discontinuous motion of particles. It is a further and also harder question what the precise laws are, e.g. whether the wave function undergoes a stochastic and nonlinear collapse evolution¹⁷.

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¹⁷It has been argued that protective measurement and the picture of random discontinuous motion of particles seem to support the reality of wavefunction collapse, and dynamical collapse theories are in the right direction in solving the measurement problem (Gao 2013b).

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